EE 359 - Wireless Communications - Fall 2009

Homework 5 Solutions

1. (6-13) (15pts)

   (a) (3pts)
   Found out d_{min} geometrically, $d_{min} = 2 \times \sqrt{E_s}\sin(\frac{\pi}{16}) = 0.3902\sqrt{E_s}$.

   (b) (3pts) $P_s = \alpha_m Q\left(\sqrt{\beta_m \gamma_s}\right) = 2Q\left(\sqrt{\frac{{d_{min}}^2}{2N_o}}\right) = 2Q(\sqrt{.076\gamma_s})$
   $\alpha_m = 2$, $\beta_m = .076$

   (c) (3pts) $\overline{P_e} = \int_0^\infty P_s(\gamma_s)f(\gamma_s)d\gamma_s$
   $= \int_0^\infty \alpha_m Q(\sqrt{\beta_m \gamma_s})f(\gamma_s)d\gamma_s$
   Using alternative Q form
   $= \frac{\alpha_m}{\pi}\int_0^{\frac{\pi}{2}}\left(1 + \frac{g\overline{\gamma_s}}{(\sin\phi)^2}\right)^{-1}d\phi$ with g $= \frac{\beta_m}{2}$
   $= \frac{\alpha_m}{2}\left[1 - \sqrt{\frac{g\overline{\gamma_s}}{1+g\overline{\gamma_s}}}\right] = 1 - \sqrt{\frac{.038\overline{\gamma_s}}{1+.038\overline{\gamma_s}}} = \frac{1}{.076\overline{\gamma_s}}$, where we have used an integral table to evaluate the integral

   (d) (3pts) $P_d = \frac{P_s}{4}$

   (e) (3pts) BPSK: $\overline{P}_b = \frac{1}{4\overline{\gamma}_b} = 10^{-3}$, $\Rightarrow \overline{\gamma_b} = 250$, 16PSK: From above get $\overline{\gamma_s} = 3289.5$
   Penalty $= \frac{3289.5}{250} = 11.2$dB
   Also will accept $\gamma_b(16PSK) = 822 \Rightarrow= 5.2dB$

2. (6-20) (10pts)
   Data rate = 40 Kbps
   Since DQPSK has 2 bits per symbol. $\therefore T_s = \frac{2}{40\times10^3} = 5\times10^{-5}sec$

   DQPSK
   Using Table 6.2, Gaussian Doppler power spectrum, $\rho_c = e^{-(\pi B_D T)^2}$
   $B_D = 80Hz$
   Rician fading $K = 2$
   $\rho_c = 0.9998$ (5pts)
   Use (6.92):

$$\overline{P}_{floor} = \frac{1}{2}\left[1 - \sqrt{\frac{(\rho_c/\sqrt{2})^2}{1-(\rho_c/\sqrt{2})^2}}\right]\exp\left[-\frac{(2-\sqrt{2})K/2}{1-\rho_c/\sqrt{2}}\right] = 2.138\times10^{-5} \quad (5pts)$$

3. (6-21) (10pts)
   (5pts for the upper bound, and 5 pts for the lower bound).

   <u>ISI:</u>

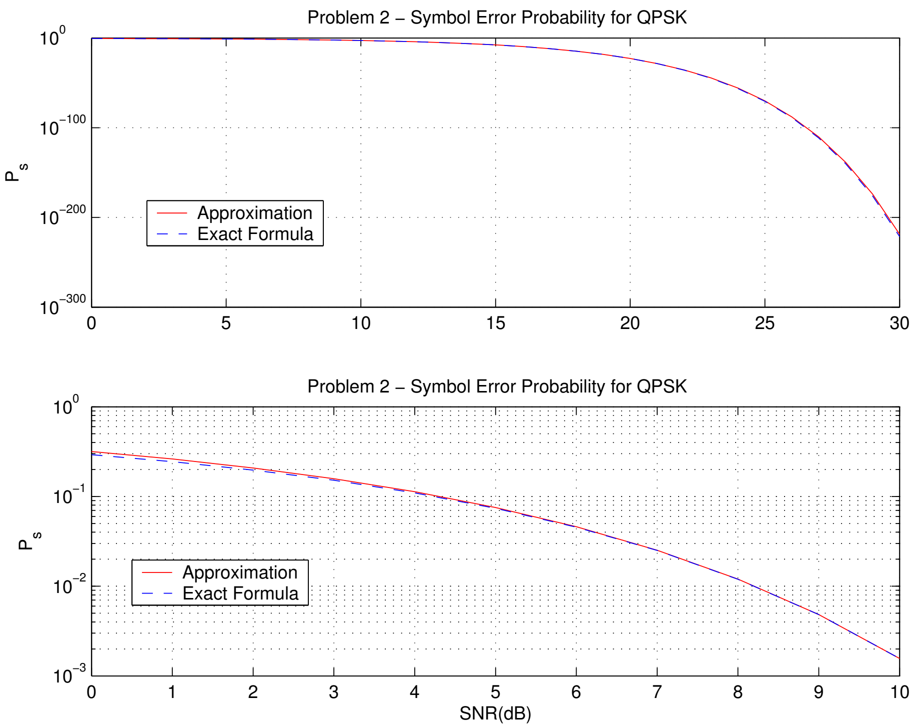

Figure 1: Problem 6-7

Thumb - Rule approach:

$\mu_t = 100$ nsec will determine ISI. As long as $T_s \gg \mu_T$, ISI will be negligible. Let $T_s \geq 10\ \mu_T$. Then $R \leq \frac{2bits}{symbol}\frac{1}{T_s}\frac{symbols}{sec} = 2$Mbps

(Will also accept: Let $T_s \geq 100\ \mu_T$. Then $R \leq \frac{2bits}{symbol}\frac{1}{T_s}\frac{symbols}{sec} = 0.2$ Mbps)

Doppler:

$B_D = 80$ Hz

$$P_{floor} = 10^{-4} \geq \frac{1}{2}\left[1 - \sqrt{\frac{\left(\rho_c/\sqrt{2}\right)^2}{1-\left(\rho_c/\sqrt{2}\right)^2}}\right]$$

$$\Rightarrow \rho_c \geq 0.9999$$

But ρ_c for uniform scattering is $J_0(2\pi B_D T_s)$, so

$$\rho_c = J_0(2\pi B_D T_s) = 1 - (\pi f_D T_s)^2 \geq 0.9999$$

$$\Rightarrow T_s \leq 39.79\mu s$$

$T_b \leq 19.89\mu s$. $R_b \geq 50.26$ Kbps.

Combining the two, we have 50.26 Kbps $\leq R_b \leq 2$ Mbps (or 0.2Mbps).

Will also accept: using expression similar to (6.93) but for DQPSK: $\bar{P}_{floor} \approx 0.5(\pi f_D T_s)^2$ so $T_s \leq \frac{\sqrt{2P_{b,target}}}{\pi B_d} = \frac{\sqrt{2\times10^{-4}}}{\pi 80} = 5.6270 \times 10^{-5}$s. So $T_b \leq 5.6270 \times 10^{-5}/2 = 2.8135 \times 10^{-5}$s, and $R \geq \frac{1}{T_b} =$ 35.5431kbps.

Combining the two, we have 35.5431 Kbps $\leq R_b \leq 2$ Mbps (or 0.2Mbps).

4. (7-1) (10pts)

$P_s = 10^{-3}$

QPSK, using (6.12), $P_s = 2Q(\sqrt{\gamma_s}) \leq 10^{-3}$, $\gamma_s \geq \gamma_0 = 10.8276$. (1pts)

$$P_{out}(\gamma_0) = \prod_{i=1}^{M} \left(1 - e^{-\frac{\gamma_0}{\overline{\gamma}_i}}\right)$$

$\overline{\gamma}_1 = 10$, $\overline{\gamma}_2 = 31.6228$, $\overline{\gamma}_3 = 100$.

$M = 1$

$P_{out} = \left(1 - e^{-\frac{\gamma_0}{\overline{\gamma}_1}}\right) = 0.6613$ (3pts)

$M = 2$

$P_{out} = \left(1 - e^{-\frac{\gamma_0}{\overline{\gamma}_1}}\right)\left(1 - e^{-\frac{\gamma_0}{\overline{\gamma}_2}}\right) = 0.1917$ (3pts)

$M = 3$

$P_{out} = \left(1 - e^{-\frac{\gamma_0}{\overline{\gamma}_1}}\right)\left(1 - e^{-\frac{\gamma_0}{\overline{\gamma}_2}}\right)\left(1 - e^{-\frac{\gamma_0}{\overline{\gamma}_3}}\right) = 0.0197$ (3pts)

5. (7-8) (15pts)

   (a) (7pts)
   $\gamma_i = 10$ dB $= 10$, $1 \leq i \leq N$
   $N = 1$, $\gamma = 10$, $M = 4$
   $P_b = .2e^{-1.5\frac{\gamma}{(M-1)}} = .2e^{-15/3} = 1.3476 \times 10^{-3}$.

   (b) (8pts)
   In MRC, $\gamma_\Sigma = \gamma_1 + \gamma_2 + \ldots + \gamma_N$.
   So $\gamma_\Sigma = 10N$

$$P_b = .2e^{-1.5\frac{\gamma_\Sigma}{(M-1)}} = .2e^{-5N} \leq 10^{-6}$$

$$\Rightarrow N \geq 2.4412$$

   So, take $N = 3$, $P_b = 6.12 \times 10^{-8} \leq 10^{-6}$.

6. (7-12)(15pts)

Average BER for BPSK, over Rayleigh fading, for two branches:

For MRC, use (7.20):

$$\bar{p}_b = \left(\frac{1-\Gamma}{2}\right)^2\left[1 + \left(1 + \frac{\Gamma}{2}\right)\right] = \left(\frac{1-\Gamma}{2}\right)^2\left(2 + \frac{\Gamma}{2}\right).$$

where $\Gamma = \frac{\bar{\gamma}}{1+\bar{\gamma}}$. (5pts for plotting the correct curve for MRC.)

For EGC, use (7.28):

$$\bar{p}_b = 0.5\left(1 - \sqrt{1 - \left(\frac{1}{1+\bar{\gamma}}\right)^2}\right).$$

(5pts for plotting the correct curve for EGC.) Plot these two probability of error, for $\bar{\gamma}$ ranging from 0 - 20 dB, shown in Fig. 2:

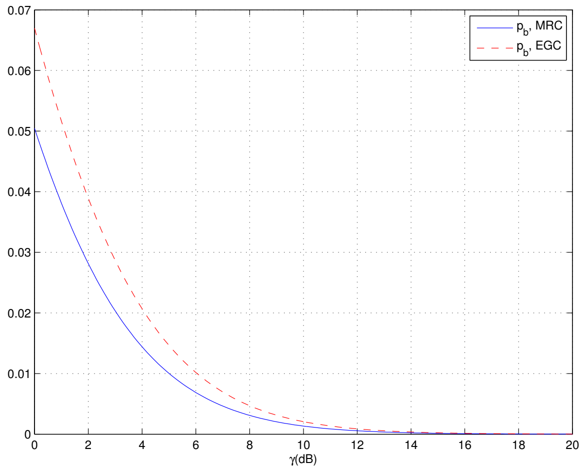

Figure 2: Problem 6-7

The maximum dB penalty to achieve the same $\bar{p}_b$ happens at $\bar{p}_b = 0.05$. The penalty is about 1.1dB. (5pts)

MATLAB code:

```
gamma_bar_dB = [0:0.1:20]; gamma_bar = 10.^(gamma_bar_dB/10);
Gamma = sqrt(gamma_bar./(1+gamma_bar)); pb_MRC =
((1-Gamma)/2).^2.*(2+Gamma/2); pb_EGC =
0.5*(1-sqrt(1-(1./(1+gamma_bar)).^2)); figure; plot(gamma_bar_dB,
pb_MRC) hold on; plot(gamma_bar_dB, pb_EGC, 'r--') legend('p_b,
MRC', 'p_b, EGC') xlabel('\gamma(dB)') grid on print -depsc
prob7_12.eps
```

7. (7-16)(10pts)

(a)(7 pts), 2 branch MRC, $p(\gamma)$ where $\int_0^\infty p(\gamma)e^{-x\gamma}d\gamma = \frac{0.01\overline{\gamma}}{\sqrt{x}}$
we will use MGF approach

$$\overline{P}_b = \frac{1}{\pi}\int_0^{\pi/2} \Pi_{i=1}^2 M_{\gamma_i}\left(-\frac{1}{\sin^2\phi}\right)d\phi$$

$$= \frac{1}{\pi}\int_0^{\pi/2}(0.01\overline{\gamma}\sin\phi)^2 d\phi$$

$$= \frac{(0.01\overline{\gamma})^2}{4}$$

(b)(3pts) $\bar{P}_b = \frac{(0.01\times 10)^2}{4} = 0.0025$.

8. (7-17) (10pts)

$\bar{\gamma} = 15dB = 31.6228$.

Rayleigh case (5pts), using (7.20):

$$\overline{P}_b = \left(\frac{1-\Gamma}{2}\right)^3 \sum_{m=0}^{2}\left(\begin{array}{c} l+m \\ m \end{array}\right)\left(\frac{1+\Gamma}{2}\right)^m; \quad \Gamma = \sqrt{\frac{\overline{\gamma}}{1+\overline{\gamma}}}$$

$\bar{p}_b = 4.5469 \times 10^{-6}$. Will also accept MGF approach:

$$\bar{p}_b = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left[1 + \frac{\bar{\gamma}}{\sin^2 \phi} \right]^{-3} d\phi = 4.5469 \times 10^{-6}.$$

Nakagami-2 fading case (5pts),

$$M_\gamma \left(-\frac{1}{\sin^2 \phi} \right) = \left(1 + \frac{\overline{\gamma}}{2 \sin^2 \phi} \right)^{-2}$$

$$\overline{P}_b = \frac{1}{\pi} \int_0^{\pi/2} \left(M_\gamma \left(-\frac{1}{\sin^2 \phi} \right) \right)^3 d\phi, \overline{\gamma} = 10^{1.5} = 5.12 \times 10^{-9}$$

```
MATLAB CODE:

gammab = 10^(1.5);
Gamma = sqrt(gammab./(gammab+1));
sumf = 0;

for m = 0:2
    f = factorial(2+m)/(factorial(2)*factorial(m));
    sumf = sumf+f*((1+Gamma)/2)^m;
end
pb_rayleigh = ((1-Gamma)/2)^3*sumf;
phi = [0.001:.001:pi/2];
sumvec = (1+(gammab./(2*(sin(phi).^2)))).^(-6);
pb_nakagami = (1/pi)*sum(sumvec)*.001;
```