EE 359 - Wireless Communications - Fall 2009

Homework 4 Solutions

1. (4-8) (9pts)

   (a) (3pts) If neither transmitter nor receiver knows when the interferer is on, they must transmit assuming worst case, i.e. as if the interferer was on all the time,

   $$C = B \log\left(1 + \frac{\overline{S}}{N_0 B + \overline{I}}\right) = 10.7Kbps.$$

   (b) (3pts) Suppose we transmit at power S_1 when jammer is off and S_2 when jammer is off,

   $$C = B \max\left[\log\left(1 + \frac{S_1}{N_o B}\right) 0.75 + \log\left(1 + \frac{S_2}{N_o B + \overline{I}}\right) 0.25\right]$$

   subject to

   $$0.75 S_1 + 0.25 S_2 = \overline{S}.$$

   This gives $S_1 = 12.25mW$, $S_2 = 3.25mW$ and $C = 53.21Kbps$.

   (c) (3pts) The jammer should transmit $-x(t)$ to completely cancel off the signal.

2. (4-13) (20pts)

   (a) (10pts) C=13.98Mbps

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MATLAB

Gammabar = [1 .5 .125]; ss = .001; P = 30e-3; N0 = .001e-6;

Bc = 4e6; Pnoise = N0*Bc; hsquare = [ss:ss:10*max(Gammabar)]; gamma
= hsquare*(P/Pnoise);

for i = 1:length(Gammabar)
    pgamma(i,:) = (1/Gammabar(i))*exp(-hsquare/Gammabar(i));
end

gamma0v = [1:.01:2]; for j = 1:length(gamma0v)
    gamma0 = gamma0v(j);
    sumP(j) = 0;
    for i = 1:length(Gammabar)
        a = gamma.*(gamma>gamma0);
        [b,c] = max(a>0);
        gammac = a(find(a));
        pgammac = pgamma(i,c:length(gamma));
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        Pj_by_P = (1/gamma0)-(1./gammac);
        sumP(j) = sumP(j) + sum(Pj_by_P.*pgammac)*ss;
    end
end [b,c] = min(abs((sumP-1))); gamma0ch = gamma0v(c);

C = 0; for i = 1:length(Gammabar)
    a = gamma.*(gamma>gamma0ch);
    [b,c] = max(a>0);
    gammac = a(find(a));
    pgammac = pgamma(i,c:length(gamma));
    C = C + Bc*ss*sum(log2(gammac/gamma0ch).*pgammac);
end
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(b) (10pts) C=13.27Mbps

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MATLAB

Gammabarv = [1 .5 .125]; ss = .001; Pt = 30e-3; N0 = .001e-6;

Bc = 4e6; Pnoise = N0*Bc; P = Pt/3; for k = 1:length(Gammabarv)
    Gammabar = Gammabarv(k);
    hsquare = [ss:ss:10*Gammabar];
    gamma = hsquare*(P/Pnoise);
    pgamma = (1/Gammabar)*exp(-hsquare/Gammabar);
    gamma0v = [.01:.01:1];
    for j = 1:length(gamma0v)
        gamma0 = gamma0v(j);
        a = gamma.*(gamma>gamma0);
        [b,c] = max(a>0);
        gammac = a(find(a));
        pgammac = pgamma(c:length(gamma));
        Pj_by_P = (1/gamma0)-(1./gammac);
        sumP(j) = sum(Pj_by_P.*pgammac)*ss;
    end
    [b,c] = min(abs((sumP-1)));
    gamma0ch = gamma0v(c);
    a = gamma.*(gamma>gamma0ch);
    [b,c] = max(a>0);
    gammac = a(find(a));
    pgammac = pgamma(c:length(gamma));
    C(k) = Bc*ss*sum(log2(gammac/gamma0ch).*pgammac);
end Ctot = sum(C);
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3. (6-8) (9pts)

(a) (2pts)

$$I_x(a) = \int_0^\infty \frac{e^{-at^2}}{x^2 + t^2} dt$$

since the integral converges we can interchange integral and derivative for a¿0

$$\begin{aligned} \frac{\partial I_x(a)}{\partial a} &= \int_0^\infty \frac{-te^{-at^2}}{x^2+t^2}dt \\ x^2 I_x(a) - \frac{\partial I_x(a)}{\partial a} &= \int_0^\infty \frac{(x^2+t^2)e^{-at^2}}{x^2+t^2}dt = \int_0^\infty e^{-at^2}dt = \frac{1}{2}\sqrt{\frac{\pi}{a}} \end{aligned}$$

(b) (3pts) Multiply both sides of (6.98) by e^{-ax^2}, then we have

$$x^2 I_x(a)e^{-ax^2} - e^{-ax^2}\frac{\partial}{\partial a}I_x(a) = \frac{1}{2}\sqrt{\frac{\pi}{a}}e^{-ax^2}.$$

Then we can rewrite the left hand side of the equation:

$$-\frac{\partial}{\partial a}(I_x(a)e^{-ax^2}) = \frac{1}{2}\sqrt{\frac{\pi}{a}}e^{-ax^2}.$$

Hence integrate both sides with respect a:

$$I_x(a)e^{-ax^2} = -\int_{-\infty}^a \frac{1}{2}\sqrt{\frac{\pi}{y}}e^{-yx^2}dy.$$

We want to write the right hand side in the error function form, so we change the variable $z = \sqrt{y}x$, and after rearranging terms we have:

$$I_x(a)e^{-ax^2} = \frac{\pi}{2x}erfc(x\sqrt{a}).$$

Multiply both sides by e^{ax^2} we have the final result.

(c) (3pts)

$$\begin{aligned} erfc(x\sqrt{a}) &= I_x(a)\frac{2x}{\pi}e^{-ax^2} = \frac{2x}{\pi}e^{-ax^2}\int_0^\infty \frac{e^{-at^2}}{x^2+t^2}dt \\ a = 1 & \\ erfc(x) &= \frac{2x}{\pi}e^{-ax^2}\int_0^\infty \frac{e^{-at^2}}{x^2+t^2}dt \quad \text{(change of variable } t = x\frac{\cos\theta}{\sin\theta}) \\ &= \frac{2}{\pi}\int_{\frac{\pi}{2}}^0 xe^{-\frac{x^2}{\sin\theta}}\frac{\sin^2\theta}{x^2}x(-\frac{1}{\sin^2\theta})d\theta \quad (dt = x(-\frac{1}{\sin^2\theta})d\theta) \\ &= \frac{2}{\pi}\int_0^{\pi/2} e^{-x^2/sin^2\theta}d\theta \end{aligned}$$

(d) (1pt)

$$Q(x) = \frac{1}{2}erfc(x/\sqrt{2}) = \frac{1}{\pi}\int_0^{\pi/2} e^{-x^2/2sin^2\theta}d\theta$$

4. (6-10) (6pts)
$T_s = 15\mu sec$
at 1mph $T_c = \frac{1}{B_d} = \frac{1}{v/\lambda} = 0.74s \gg T_s$
$\therefore$ outage probability is a good measure. (2pts)
at 10 mph $T_c = 0.074s \gg T_s$ $\therefore$ outage probability is a good measure. (2pts)
at 100 mph $T_c = 0.0074s = 7400\mu s > 15\mu s$ outage or outage combined with average prob of error can be a good measure. (2pts)

5. (6-12) (12pts)

(a) (6pts) When there is path loss alone, $d = \sqrt{100^2 + 500^2} = 100\sqrt{26} \times 10^3 = 509.9$km
$P_e = \frac{1}{2}e^{-\gamma_b} \Rightarrow \gamma_b = 13.1224$
$\lambda = \frac{c}{f} = 1/3(m)$
$\frac{P_r}{N_0 B} = 13.1224 \Rightarrow P_r = 1.3122 \times 10^{-14}(W)$
$\frac{P_r}{P_t} = \left[\frac{\sqrt{G}\lambda}{4\pi d}\right]^2 \Rightarrow P_t = 4.8488W$

(b) (6pts) The required received power for 10^{-6}BER is

$$x = 1.3122 \times 10^{-14}(W) = 1.3122 \times 10^{-11}mW = -108.82dBm$$

This is the target received power. Then we know the actual received power due to shadowing in dB is normal distributed

$$P_{\gamma,dB} \sim N(\mu, 8^2), \sigma_{dB} = 8$$

To have an outage probability 0.9, we require

$$\begin{aligned} P(P_{\gamma,dB} \geq x) &= 0.9 \\ P\left(\frac{P_{\gamma,dB} - \mu}{8} \geq \frac{x-\mu}{8}\right) &= 0.9 \\ \Rightarrow Q\left(\frac{x-\mu}{8}\right) &= 0.9 \\ \Rightarrow \frac{x-\mu}{8} &= Q^{-1}(0.9) = -1.2816 \\ \Rightarrow \mu = -98.5672dBm = 1.3908 \times 10^{-10}mW \end{aligned}$$

This μ is the received power due to free space pathloss, so

$$P_r = P_t[\frac{\lambda}{4\pi d}]^2 \rightarrow P_t = 51.3940W.$$

6. (6-16) (8pts)
For DPSK in Rayleigh fading, $\overline{P_b} = \frac{1}{2\overline{\gamma_b}} \Rightarrow \overline{\gamma_b} = 500$ (1pt)
$N_o B = 3 \times 10^{-12} mW \Rightarrow P_{target} = \overline{\gamma}_b N_0 B = 1.5 \times 10^{-9} mW$ = -88.24 dBm (1pt)

Now, consider shadowing. We also want to keep the outage probability to be 0.01
$P_{out} = P[P_r < P_{target}] = P[\frac{P_r - \overline{P_r}}{\sigma} < \frac{P_{target} - \overline{P_r}}{\sigma}] = \Phi\left(\frac{P_{target} - \overline{P_r}}{\sigma}\right) \triangleq 0.01$ (1pt)
$\Rightarrow \Phi^{-1}(.01) = 2.327 = \frac{P_{target} - \overline{P_r}}{\sigma}$
$\overline{P_r} = -74.28$ dBm $= 3.7342 \times 10^{-8}$ mW(3pt)
$\overline{P_r} = P_t\left(\frac{\lambda}{4\pi d}\right)^2 = 100(mW)\left(\frac{\lambda}{4\pi d}\right)^2$
$\lambda = 1/3m, \Rightarrow$ d = 1372.7 m (2pt)

7. (6-17) (6pts)

(a) (2pts)
From simplified path-loss model:

$$\begin{aligned} P_r &= P_t k (\frac{d_0}{d})^\gamma \\ P_r(dBm) &= 20dBm - 30\log_{10}(100)(dB) = -40dBm \end{aligned}$$

(b) (2pts)

$$\bar{P}_b \approx \frac{1}{4\bar{\gamma}_b}$$
$$\Rightarrow \bar{\gamma}_b \geq \frac{1}{4\bar{P}_b} = 2.5 \times 10^3$$

$$\bar{p}_{min} = \bar{\gamma}_b N_0 B = 7.5 \times 10^{-7} = -31.25dBm$$

(c) (2pts)

$$c = Q(a) + \exp \frac{2 - 2ab}{b^2} Q\left(\frac{2 - ab}{b}\right) = 31\%$$

where

$$a = \frac{\bar{P}_{min} - \bar{P}_r(R)}{\sigma_{\psi dB}} = 2.1875$$
$$b = \frac{10\gamma \log e}{\sigma_{\psi dB}} = 3.2572$$