EE 359 - Wireless Communications - Fall 2009

Homework 2 Solutions

1. (2-18)

   (a) (6 pts) (Approach I): $d0 = 1$
   $K = 20\log_{10}(\frac{\lambda}{4\pi d_0}) = 20\log_{10}(\frac{1/3}{4*\pi}) = -31.54dB$. (3pts)
   The model: $M_{model}(d_i) = K - 10\gamma 10\log_{10}(d_i)$.
   $F(r) = \frac{1}{6}\sum_{i=1}^{6}[M_{measured}(d_i) - M_{model}(d_i)]^2 = \frac{1}{6}\sum_{i=1}^{6}[M_{measured}(d_i) - K + 10\gamma 10\log_{10}(d_i)]^2$
   Now use vector representation $\mathbf{M} = [M_{measured}(d_1), \cdots M_{measured}(d_6)]^T - K$, where T is transpose. Also define $\mathbf{d}(dB) = [10\log_{10}(d_1), \cdots 10\log_{10}(d_6)]^T$. These are both 6 dimensional vectors. Now we can use convenient representation:

   $$F(\gamma) = \frac{1}{6}\|\mathbf{M} - \gamma\mathbf{d}\|^2,$$

   where $\|\mathbf{x}\|^2 = \mathbf{x}^T\mathbf{x}$ is the L2 norm of $\mathbf{x}$. Now to find γ that minimizes $F(\gamma)$, find the derivative: $\frac{\partial F(\gamma)}{\partial\gamma} = \frac{1}{3}(\mathbf{M} + \gamma\mathbf{d})^T\mathbf{d}$. Set it to 0, then we find $\gamma = -\frac{\mathbf{M}^T d}{\mathbf{d}^T\mathbf{d}} = 3.9581$. (To here 7 pts.)
   Matlab code for approach I

```
didB = 10*log10([5 25 65 110 400 1000]’); Mm = [-60 -80 -105 -115
-135 -150]’; K = 20*log10(1/3/4/pi); gamma = -(Mm -
K)’*(didB)/(didB’*didB)
```

   (Approach II): you can also use method provided in example 2.3 of the text, which does not use vector representation. Result is the same: $\gamma = 3.9581$.
   The variance of the log-normal shadowing about the mean path loss based on the measurements: $\sigma^2_{\psi_{dB}} = \sigma^2_{\psi_{dB}} = F(3.9581) = 9.6810$

   (b) (2pts) The path loss at 2km by this model is: $K - 10\gamma\log_{10}(\frac{d}{d_0}) = -31.54 - 10 * 3.9581 * \log_{10}(2000) = -162.1981$dB.

   (c) (2pts)

   $$\begin{aligned} P_{out} &= P(P_{FreeSpace} + \psi_{dB} < P_{req}) \\ &= P(\psi_{dB} < P_{req} - P_{FreeSpace}) \quad [P_{FreeSpace} - P_{req} = 10dB] \\ &= 1 - Q(\frac{-10}{\sigma_{\psi_{dB}}}) \\ &= 6.5462 \times 10^{-4}. \end{aligned}$$

2. (3-1) (10 pts)
   $d = vt$
   $r + r' = 2\sqrt{h^2 + (\frac{d}{2})^2} = 2(\frac{d}{2})\sqrt{1 + (\frac{2h}{d})^2}$
   $(d \gg h, \rightarrow \frac{2h}{d} \ll 1)$ Use Taylor expansion $\sqrt{1+x} \approx 1 + \frac{1}{2}x$ $r + r' \approx d(1 + \frac{1}{2}(\frac{2h}{d})^2) = d + \frac{2h^2}{d}$ (this is where the result in hint comes from).
   Equivalent low-pass channel impulse response is given by

   $$c(\tau, t) = \alpha_0(t)e^{-j\phi_0(t)}\delta(\tau - \tau_0(t)) + \alpha_1(t)e^{-j\phi_1(t)}\delta(\tau - \tau_1(t))$$

$\alpha_0(t) = \frac{\lambda\sqrt{G_l}}{4\pi d}$ with $d = vt$
$\phi_0(t) = 2\pi f_c\tau_0(t) - \phi_{D_0} = 2\pi f_c\frac{vt}{c} + 2\pi\frac{vt}{\lambda} = 4\pi f_c\frac{vt}{c} \quad (\frac{f_c}{c} = \lambda)$
$\tau_0(t) = d/c$
$\phi_{D_0} = \int_t 2\pi f_{D_0}(t)dt$
$f_{D_0}(t) = \frac{v}{\lambda}\cos\theta_0(t) = -\frac{2\pi v}{\lambda}t,$
$\theta_0(t) = \pi\ \forall t$
$\alpha_1(t) = \frac{\lambda R\sqrt{G_l}}{4\pi(r+r')} = \frac{\lambda R\sqrt{G_l}}{4\pi(d+\frac{2h^2}{d})}$ with $d = vt$
$\phi_1(t) = 2\pi f_c\tau_1(t) - \phi_{D_1}$
$\tau_1(t) = (r + r')/c = (d + \frac{2h^2}{d})/c$
$f_{D_1}(t) = \frac{v}{\lambda}\cos\theta_1(t)$
$\phi_{D_1} = \int_t 2\pi f_{D_1}(t)dt$
$\cos\theta_1(t) = -\frac{d}{2r'} = -\frac{1}{(1+\frac{2h^2}{d^2})}\forall t$

3. (3-5) (10 pts) (each part 5 pts.)
Use CDF strategy.

$$F_z(z) = P[x^2+y^2 \le z^2] = \int\int_{x^2+y^2\le z^2} \frac{1}{2\pi\sigma^2}e^{\frac{-(x^2+y)^2}{2\sigma^2}}dxdy = \int_0^{2\pi}\int_0^{z} \frac{1}{2\pi\sigma^2}e^{\frac{-r^2}{2\sigma^2}}rdrd\theta = 1 - e^{\frac{-z^2}{2\sigma^2}}(z \ge 0)$$

$$\frac{df_z(z)}{dz} = \frac{z}{\sigma^2}e^{-\frac{z^2}{2\sigma^2}} \rightarrow Rayleigh$$

For Power:

$$F_{z^2}(z) = P[Z \le \sqrt{z}] = 1 - e\frac{-z^2}{2\sigma^2}$$

$$f_z(z) = \frac{1}{2\sigma^2}e\frac{-z}{2\sigma^2} \rightarrow Exponential$$

4. (3-7) (3 pts)
For Rayleigh fading channel

$$P_{outage} = 1 - e^{-P_0/2\sigma^2}$$

$$0.01 = 1 - e^{-P_0/P_r}$$

$$\therefore P_r = -60dBm$$

5. (3-8) (7 pts)
$2\sigma^2$ = -80dBm = $10^{-11}W$
Target Power P_0 = -80 dBm = $10^{-11}W$
Avg. power in LOS component = s^2 = -80dBm = $10^{-11}W$ (4 pts for change-of-units)

$$Pr[z^2 \le 10^{-11}] = Pr[z \le \frac{10^{-5}}{\sqrt{10}}]$$

Let $z_0 = \frac{10^{-5}}{\sqrt{10}}$

$$= \int_0^{z_0} \frac{z}{\sigma^2}e^{\left[-\frac{-(z^2+s^2)}{2\sigma^2}\right]}I_0\left(\frac{zs}{\sigma^2}\right)dz, \quad z \ge 0$$

$$= 0.3457$$

(3 pts for getting a roughly correct final answer.) To evaluate this, we use Matlab and $I_0(x)$ = besseli(0,x). Sample Code is given:

```
clear P0 = 1e-11; s2 = 1e-11; sigma2 = (1e-11)/2; z0 = sqrt(1e-11);
ss = z0/1e7; z = [0:ss:z0]; pdf =
(z/sigma2).*exp(-(z.^2+s2)/(2*sigma2)).*besseli(0,z.*(sqrt(s2)/sigma2));
int_pr = sum(pdf)*ss;
```

6. (3-9) (10pts)
CDF of Ricean distribution is

$$F_Z^{\text{Ricean}}(z) = \int_0^z p_Z^{\text{Ricean}}(z)$$

where

$$p_Z^{\text{Ricean}}(z) = \frac{2z(K+1)}{Pr}\exp\left[-K - \frac{(K+1)z^2}{Pr}\right] I_0\left(2z\sqrt{\frac{K(K+1)}{Pr}}\right), \quad z \geq 0$$

For the Nakagami-m approximation to Ricean distribution, we set the Nakagami m parameter to be $(K+1)^2/(2K+1)$. CDF of Nakagami-m distribution is

$$F_Z^{\text{Nakagami-m}}(z) = \int_0^z p_Z^{\text{Nakagami-m}}(z)$$

where

$$p_Z^{\text{Nakagami-m}}(z) = \frac{2m^m z^{2m-1}}{\Gamma(m)Pr^m}\exp\left[\frac{-mz^2}{Pr}\right], \quad z \geq 0, \quad m \geq 0.5$$

(5pts for the right approach: finding CDF by integral, and getting correcting plots.)

We need to plot the two CDF curves for K = 1,5,10 and Pr =1 (we can choose any value for Pr as it is the same for both the distributions and our aim is to compare them). Sample code is given:

```
z = [0:0.01:3]; K = 10; m = (K+1)^2/(2*K+1); Pr = 1; pdfR =
((2*z*(K+1))/Pr).*exp(-K-((K+1).*(z.^2))/Pr).*besseli(0,(2*sqrt((K*(K+1))/Pr))*z);
pdfN = ((2*m^m*z.^(2*m-1))/(gamma(m)*Pr^m)).*exp(-(m/Pr)*z.^2); for
i = 1:length(z)
    cdfR(i) = 0.01*sum(pdfR(1:i));
    cdfN(i) = 0.01*sum(pdfN(1:i));
end plot(z,cdfR); hold on plot(z,cdfN,'b--'); figure; plot(z,pdfR);
hold on plot(z,pdfN,'b--');
```

As seen from the curves, the Nakagami-m approximation becomes better as K increases. Also, for a fixed value of K and x, prob(γ<x) for x large is always greater for the Ricean distribution. This is seen from the tail behavior of the two distributions in their pdf, where the tail of Nakagami-distribution is always above the Ricean distribution. (5 pts for getting this correct conclusion.)

7. (3-10) (15 pts)

   (a) (2 pts) W = average received power
   Z_i = Shadowing over link i
   P_{ri} = Received power in dBW, which is Gaussian with mean W, variance σ^2

   (b) (4 pts)

$$P_{outage} = P\left[P_{r,1} < T \cap P_{r,2} < T\right] = P\left[P_{r,1} < T\right] P[P_{r,2} < T] \; \textit{since } Z_1, Z_2 \textit{ independent}$$

$$= \left[Q\left(\frac{W-T}{\sigma}\right)\right]^2 = \left[Q\left(\frac{\triangle}{\sigma}\right)\right]^2$$

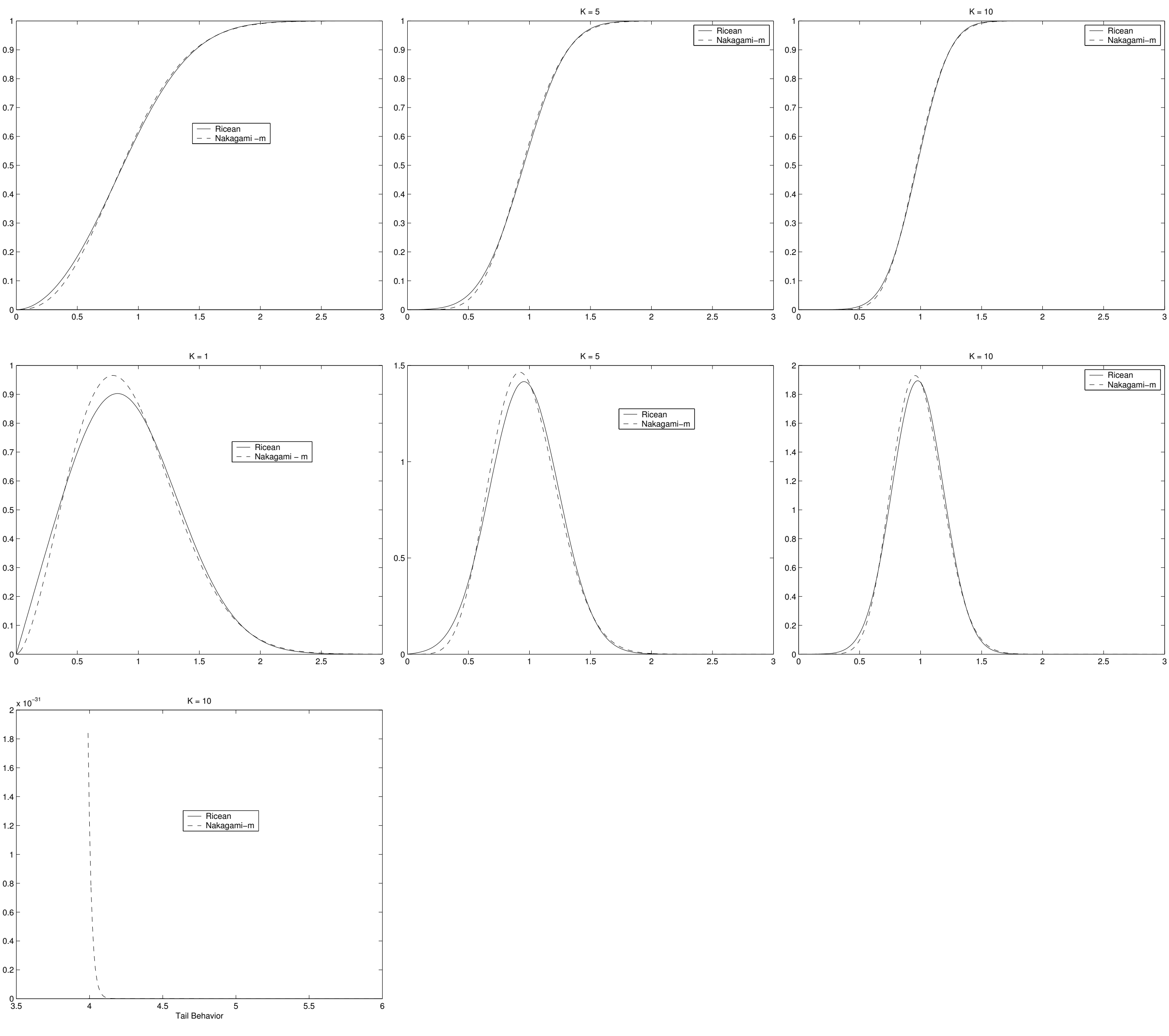

Figure 1: The CDF and PDF for $K = 1, 5, 10$ and the Tail Behavior

(c) (7 pts)

$$P_{out} = \int_{-\infty}^{\infty} P\left[P_{r,1} \leq T\ ,\ P_{r,2} < T|Y = y\right] f_y(y)\, dy$$

$$P_{r,1}|Y = y \sim N\left(W + by, a^2\sigma^2\right),\ and\ [P_{r,1}|Y = y] \perp [P_{r,2}|Y = y]$$

$$P_{outage} = \int_{-\infty}^{\infty} \left[Q\left(\frac{W + by - T}{a\sigma}\right)\right]^2 \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy$$

let $\frac{y}{\sigma} = u$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \left[Q\left(\frac{W - T + bu\sigma}{a\sigma}\right)\right]^2 e^{\frac{-u^2}{2}} du = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \left[Q\left(\frac{\triangle + by\sigma}{a\sigma}\right)\right]^2 e^{\frac{-y^2}{2}} dy$$

(d) (2 pts) Let a = b = $\frac{1}{\sqrt{2}}$, $\sigma = 8$, $\triangle = 5$. With independent fading we get

$$P_{out} = \left[Q\left(\frac{5}{8}\right)\right]^2 = 0.0708.$$

With correlated fading we get $P_{out} =$ 0.1316.
Conclusion : Independent shadowing is prefarable for diversity.

8. (3-11) (15 pts) (Getting the correct approach, have reasonable result.)
There are many acceptable techniques for this problem. Sample code for both the stochastic technique(preferred) and the Jake's technique are included.

Jakes: Summing of appropriate sine waves

```
%Jake's Method
close all; clear all;
%choose N=30
N=30; M=0.5*(N/2-1); Wn(M)=0; beta(M)=0;
%We choose 1000 samples/sec
ritemp(M,2001)=0; rqtemp(M,2001)=0; rialpha(1,2001)=0; fm=[1 10
100]; Wm=2*pi*fm; for i=1:3
    for n=1:1:M
        for t=0:0.001:2
            %Wn(i)=Wm*cos(2*pi*i/N)
            Wn(n)=Wm(i)*cos(2*pi*n/N);
            %beta(i)=pi*i/M
            beta(n)=pi*n/M;
            %ritemp(i,20001)=2*cos(beta(i))*cos(Wn(i)*t)
            %rqtemp(i,20001)=2*sin(beta(i))*cos(Wn(i)*t)
            ritemp(n,1000*t+1)=2*cos(beta(n))*cos(Wn(n)*t);
            rqtemp(n,1000*t+1)=2*sin(beta(n))*cos(Wn(n)*t);
            %Because we choose alpha=0,we get sin(alpha)=0 and cos(alpha)=1
            %rialpha=(cos(Wm*t)/sqrt(2))*2*cos(alpha)=2*cos(Wm*t)/sqrt(2)
            %rqalpha=(cos(Wm*t)/sqrt(2))*2*sin(alpha)=0
```

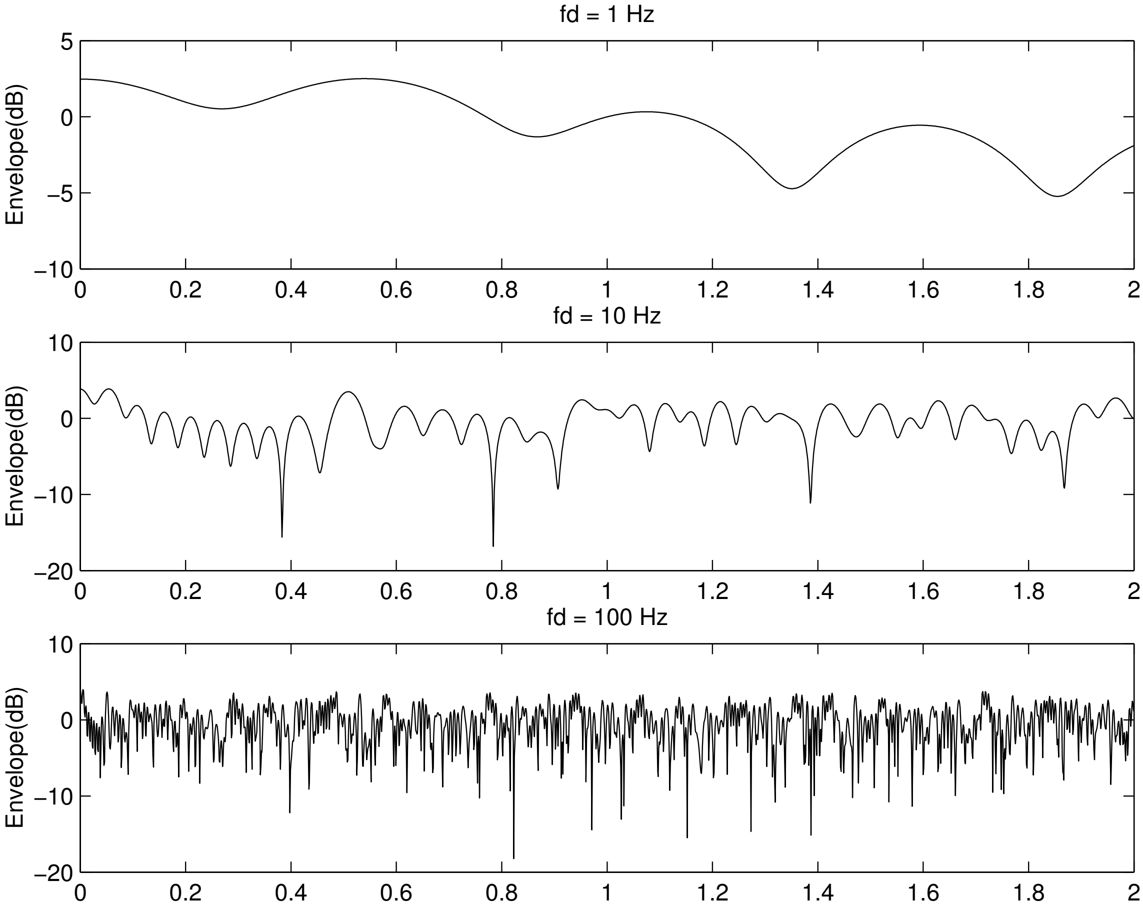

Figure 2: Problem 11

```
            rialpha(1,1000*t+1)=2*cos(Wm(i)*t)/sqrt(2);
        end
    end
    %summarize ritemp(i) and rialpha
    ri=sum(ritemp)+rialpha;
    %summarize rqtemp(i)
    rq=sum(rqtemp);
    %r=sqrt(ri^2+rq^2)
    r=sqrt(ri.^2+rq.^2);
    %find the envelope average
    mean=sum(r)/2001;
    subplot(3,1,i);
    time=0:0.001:2;
    %plot the figure and shift the envelope average to 0dB
    plot(time,(10*log10(r)-10*log10(mean)));
    titlename=['fd = ' int2str(fm(i)) ' Hz'];
    title(titlename);
    xlabel('time(second)');
    ylabel('Envelope(dB)');
end
```

Stochastic: Usually two guassian R.V.'s are filtered by the Doppler Spectrum and summed. Can also just do a Rayleigh distribution with an adequate LPF, although the above technique is prefered.

```
function [Ts, z_dB] = rayleigh_fading(f_D, t, f_s)
%
% function [Ts, z_dB] = rayleigh_fading(f_D, t, f_s)
% generates a Rayleigh fading signal for given Doppler frequency f_D,
% during the time perios [0, t], with sampling frequency f_s >= 1000Hz.
%
% Input(s)
%   -- f_D  : [Hz] [1x1 double] Doppler frequency
%   -- t    : simulation time interval length, time interval [0,t]
%   -- f_s  : [Hz] sampling frequency, set to 1000 if smaller.
% Output(s)
%   -- Ts   : [Sec][1xN double] time instances for the Rayleigh signal
%   -- z_dB : [dB] [1xN double] Rayleigh fading signal
%

% Required parameters
if f_s < 1000;
  f_s = 1000;                 % [Hz]  Minumum required sampling rate
end;
N = ceil( t*f_s );            % Number of samples

% Ts contains the time instances at which z_dB is specified
Ts = linspace(0,t,N);

if mod( N, 2) == 1
  N = N+1;                    % Use even number of samples in calculation
end;
f = linspace(-f_s,f_s,N);     % [Hz]  Frequency samples used in calculation

% Generate complex Gaussian samples with line spectra in frequency domain
% Inphase :
Gfi_p = randn(2,N/2); CGfi_p = 1/sqrt(2)*(
Gfi_p(1,:)+i*Gfi_p(2,:)); CGfi = [ conj(fliplr(CGfi_p)) CGfi_p ];

% Quadrature :
Gfq_p = randn(2,N/2); CGfq_p =
 1/sqrt(2)*(Gfq_p(1,:)+i*Gfq_p(2,:)); CGfq = [
conj(fliplr(CGfq_p)) CGfq_p ];

% Generate fading spectrum, this is used to shape the Gaussian line spectra
omega_p = 1;   % this makes sure that the average received envelop can be 0dB
S_r = omega_p/4/pi./(f_D*sqrt(1-(f/f_D).^2));

% Take care of samples outside the Doppler frequency range, let them be 0
idx1 = find(f>=f_D); idx2 = find(f<=-f_D); S_r(idx1) = 0; S_r(idx2)
= 0; S_r(idx1(1)) = S_r(idx1(1)-1); S_r(idx2(length(idx2))) =
S_r(idx2(length(idx2))+1);
```

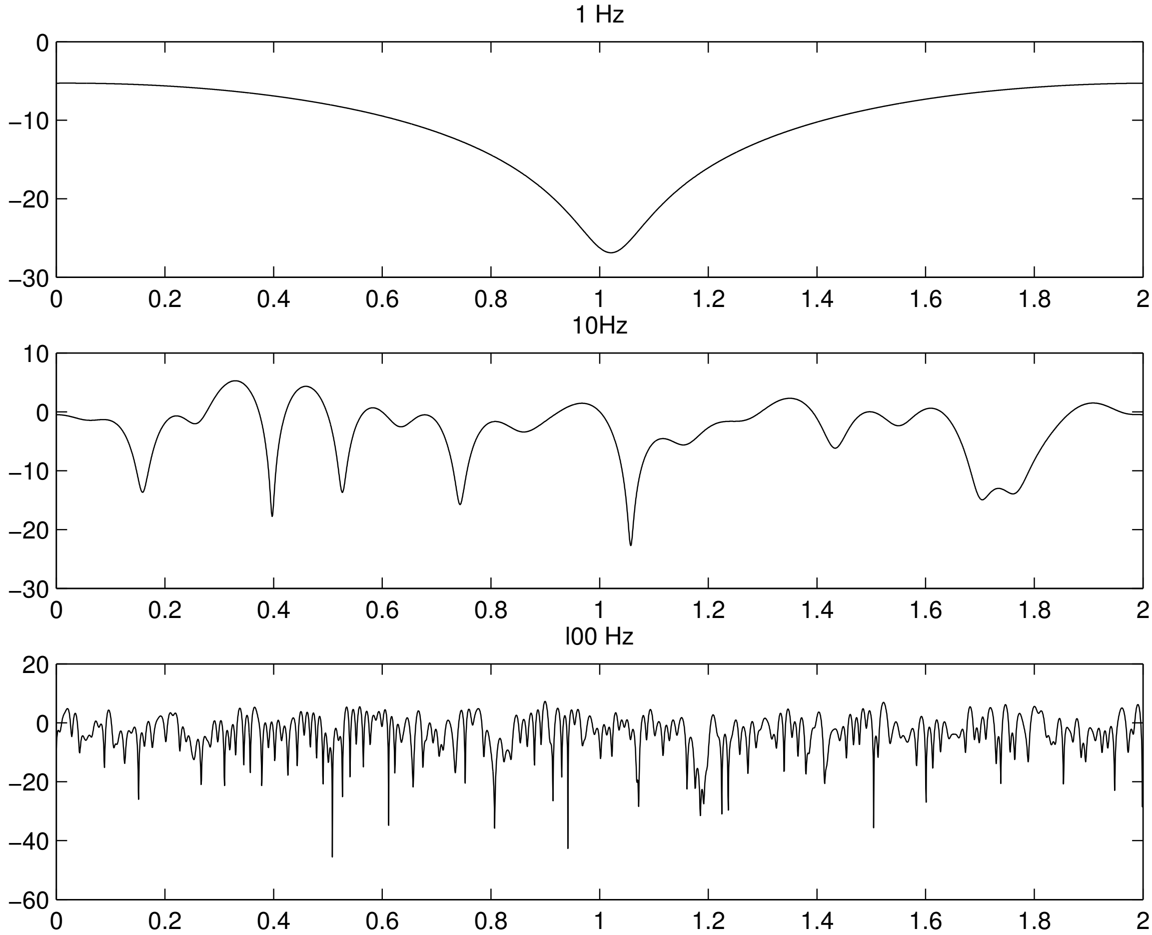

Figure 3: Problem 11

```
% Generate r_I(t) and r_Q(t) using inverse FFT:
r_I = N*ifft(CGfi.*sqrt(S_r)); r_Q = -i*N*ifft(CGfq.*sqrt(S_r));

% Finally, generate the Rayleigh distributed signal envelope
z = sqrt(abs(r_I).^2+abs(r_Q).^2); z_dB = 20*log10(z);

% Return correct number of points
z_dB = z_dB(1:length(Ts));
```