EE 359 - Wireless Communications - Fall 2009

Homework 1 Solutions

1. (1-3) (4 points)
   $P_b = 10^{-12}$
   $\frac{1}{2\overline{\gamma}} = 10^{-12}$
   $\overline{\gamma} = \frac{10^{12}}{2} = 5 \times 10^{11}$ (very high)

2. (1-6) (13 points)

   optimum no. of data user = d
   optimum no. of voice user = v
   Three different cases:
   (2 points) Case 1: d=0, v=6
   $\Rightarrow revenue = 60.80.2 = 0.96$

   (9 points) Case 2: d=1, v=3
   revenue = [prob. of having one data user]×(revenue of having one data user)
   + [prob. of having two data user]×(revenue of having two data user)
   + [prob. of having one voice user]×(revenue of having one voice user)
   + [prob. of having two voice user]×(revenue of having two voice user)
   + [prob. of having three or more voice user]×(revenue in this case)

$$\Rightarrow \quad 0.5^2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} \times \$1 + 0.5^2 \times \$1 + \begin{pmatrix} 6 \\ 1 \end{pmatrix} 0.8 \times 0.2^5 \times \$0.2 + \begin{pmatrix} 6 \\ 2 \end{pmatrix} 0.8^2 \times 0.2^4 \times \$0.4 +$$

$$\left[1 - \begin{pmatrix} 6 \\ 1 \end{pmatrix} 0.8 \times 0.2^5 \times \$0.2 - \begin{pmatrix} 6 \\ 2 \end{pmatrix} 0.8^2 \times 0.2^4 \times \$0.4\right] \times \$0.6$$

$$\Rightarrow \$1.35$$

   (2 points) Case 3: d=2, v=0
   revenue $= 2 \times 0.5 = \$1$
   So the best case is case 2, which is to allocate 60kHz to data and 60kHz to voice.

3. (1-10)(13points)

   Approach I:

   (a) (1point) 100 cells, 100 users/cell $\Rightarrow$ 10,000 users
   (b) (1point)100 users/cell $\Rightarrow$ 2500 cells required
   $\frac{100km^2}{Area/cell} = 2500cells \Rightarrow \frac{Area}{cell} = .04km^2$

(c) (7points) From Rappaport or iteration of formula, we get that 100 $\frac{channels}{cell} \Rightarrow 89 \frac{channels}{cell}$ @$P_b = .02$
Each subscriber generates $\frac{1}{30}$ of an Erlang of traffic.
Thus, each cell can support $30 \times 89 = 2670$ subscribers
Macrocell: $2670 \times 100 \Rightarrow 267,000$ subscribers
Microcell: 6,675,000 subscribers

(d) (2point) Macrocell: \$50 M
Microcell: \$1.25 B

(e) (2point) Macrocell: \$13.35 M/month $\Rightarrow$ 3.75 months *approx* 4 months to recoop
Microcell: \$333.75 M/month $\Rightarrow$ 3.75 months *approx* 4 months to recoop

Approach II: (same points break down as approach I)

(a)

(b) (a)-(b) Same as Approach I.

(c) For this setting, $C = 100$ channels, $U =$ number of users, $\mu = 1/60$, average number of call/minute/user, $H = 2$min, average duration of call. So $A = U\mu H = U/30$. To have less than 2% blocking probability during peak hour requires $P_b = \frac{A^C}{C!}\frac{1}{\sum_{k=0}^{C}\frac{A^k}{k!}} \leq 0.02 \rightarrow \frac{(\frac{U}{100})^{100}}{100!}\frac{1}{\sum_{k=0}^{100}\frac{U^k}{k!30^k}} \leq 0.02$. Use $\sum_{k=0}^{100}\frac{U^k}{k!30^k} \approx e^{U/30}$, numerically solve. The P_b is plotted in Fig. 1. Take the first root because $U = 2661.3$ (round it $U = 2661$), because if we take the second root which is larger than the first root, then for many values of U less than the first root you can have probability higher than 0.02. macrocell: 266×10^3 users, microcell: 6652.5×10^3 user

(d) same as approach I

(e) macrocell: 13.3 million/month, microcell: 332.625 million/month, they both requires about 4 months to recoup the cost.

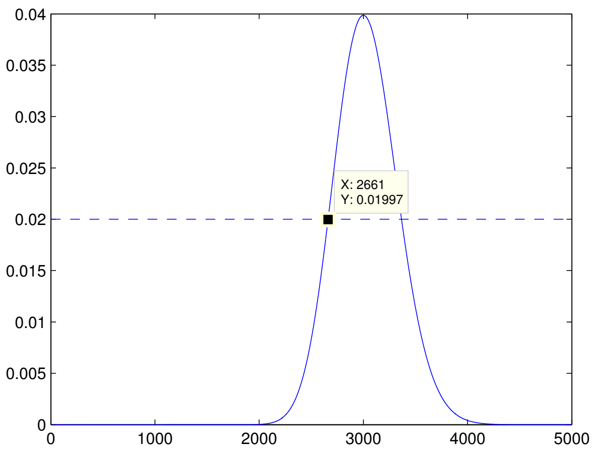

Figure 1: Problem 1-10(c)

4. (2-8) (10 points. Each case is 2.5 points.)
A program to plot the figures is shown below. The power versus distance curves and a plot of the phase difference between the two paths is shown on the following page. From the plots it can be seen that as G_r (gain of reflected path) is decreased, the asymptotic behavior of P_r tends toward d^{-2} from d^{-4}, which makes sense since the effect of reflected path is reduced and it is more like having only a

LOS path. Also the variation of power before and around dc is reduced because the strength of the reflected path decreases as G_r decreases. Also note that the the received power actually increases with distance up to some point. This is because for very small distances (i.e. d = 1), the reflected path is approximately two times the LOS path, making the phase difference very small. Since R = -1, this causes the two paths to nearly cancel each other out. When the phase difference becomes 180 degrees, the first local maxima is achieved. Additionally, the lengths of both paths are initially dominated by the difference between the antenna heights (which is 35 meters). Thus, the powers of both paths are roughly constant for small values of d, and the dominant factor is the phase difference between the paths.

```
clear all;
close all;
ht=50;
hr=15;
f=900e6;
c=3e8;
lambda=c/f;
GR=[1,.316,.1,.01];
Gl=1;
R=-1;
counter=1;
figure(1);
d=[1:1:100000];
l=(d.^2+(ht-hr)^2).^.5;
r=(d.^2+(ht+hr)^2).^.5;
phd=2*pi/lambda*(r-1);
dc=4*ht*hr/lambda;
dnew=[dc:1:100000];

for counter = 1:1:4,
    Gr=GR(counter);
    Vec=Gl./l+R*Gr./r.*exp(phd*sqrt(-1));
    Pr=(lambda/4/pi)^2*(abs(Vec)).^2;
    subplot(2,2,counter);
    plot(10*log10(d),10*log10(Pr)-10*log10(Pr(1)));
    hold on;
    plot(10*log10(dnew),-20*log10(dnew));
    plot(10*log10(dnew),-40*log10(dnew));
end
hold off
```

5. (2-14) (10points)
d = distance between cells with reused freq
p = transmit power of all the mobiles

$$\left(\frac{S}{I}\right)_{\text{uplink}} \geq 20dB$$

(a) (4 points) Min. S/I will result when main user is at A and Interferers are at B
d_A = distance between A and base station #1 = $\sqrt{2}km$, d_B = distance between B and base

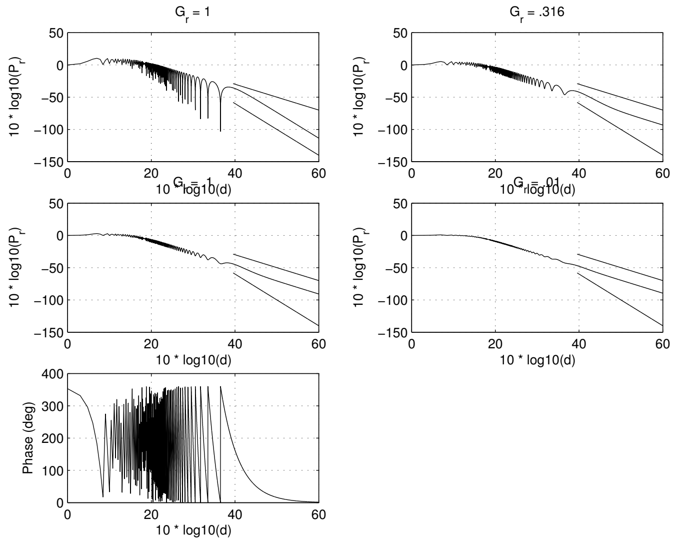

Figure 2: Problem 2-8

station #1 $= d - 1km$

$$\left(\frac{S}{I}\right)_{\min} = \frac{P\left[\frac{G\lambda}{4\pi d_A}\right]^2}{2P\left[\frac{G\lambda}{4\pi d_B}\right]^2} = \frac{d_B^2}{2d_A^2} = \frac{(d_{min}-1)^2}{4} = 100$$

$\Rightarrow d_{min} - 1 = 20km \Rightarrow d_{min} = 21km$ since integer number of cells should be accommodated in distance d $\Rightarrow d_{min} = 22km$

(b) (3 points)

$$\frac{P_\gamma}{P_u} = k\left[\frac{d_0}{d}\right]^\gamma \Rightarrow \left(\frac{S}{I}\right)_{\min} = \frac{Pk\left[\frac{d_0}{d_A}\right]^\gamma}{2Pk\left[\frac{d_0}{d_B}\right]^\gamma} =$$

$$\frac{1}{2}\left[\frac{d_B}{d_A}\right]^\gamma = \frac{1}{2}\left[\frac{d_{min}-1}{\sqrt{2}}\right]^\gamma = \frac{1}{2}\left[\frac{d_{min}-1}{\sqrt{2}}\right]^3 = 100$$

$\Rightarrow d_{min} = 9.27 \Rightarrow$ with the same argument $\Rightarrow d_{min} = 10km$

(c) (3points)

$$\left(\frac{S}{I}\right)_{\min} = \frac{k\left[\frac{d_0}{d_A}\right]_A^\gamma}{2k\left[\frac{d_0}{d_B}\right]_B^\gamma} = \frac{(d_{min}-1)^4}{0.04} = 100$$

$\Rightarrow d_{min} = 2.41km \Rightarrow$ with the same argument $d_{min} = 4km$

6. (2-16) (10 points)
Piecewise linear model for 2-path model. See Fig 3 (Having right breaking points (h_t and d_c) worth 5 points. Another 5 points for plotting the correct figure (3 pts) based on (2.12) and finding a reasonable slopes (not necessarily the exact slopes given in the Figure) (2pts).)

7. (2-19) (10 points)
Assume free space path loss parameters
$f_c = 900MHz \rightarrow \lambda = 1/3m$
$\sigma_{\psi dB} = 6$
$SNR_{recd} = 15dB$

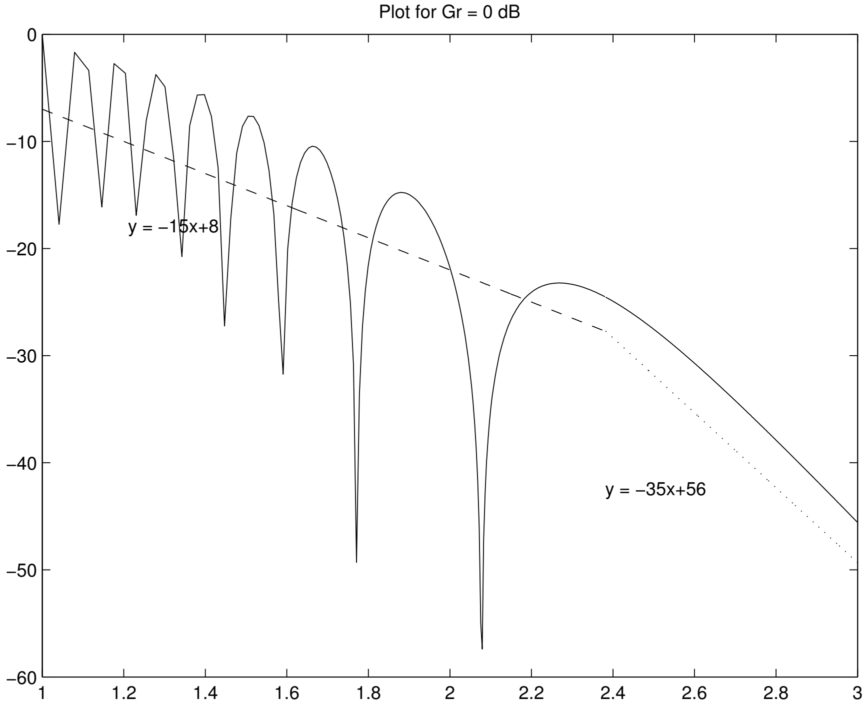

Figure 3: Problem 2-16

$P_t = 1W$
$g = 3dB$
$P_{noise} = -40dBm \Rightarrow P_{recvd} = NoisePower + SNR = -40dBm + 15dB = -25dBm$(1point)
Suppose we choose a cell of radius d

$$\begin{aligned} \mu(d)(dBm) &= 10\log_{10}[P_{recvd}(\text{due to path loss alone})] \\ &= P_t(dBm) + 10\log_{10}\left(\frac{\sqrt{G_l}\lambda}{4\pi d}\right)^2 \\ &= 30dBm + 10\log_{10}[\frac{1.4\times 10^{-3}}{d^2}](4points) \end{aligned}$$

The actual received power at d: $P_{recd}(d)$ has mean $\mu(d)$ dBm and standard deviation $\sigma_{\psi_{dB}} = 6$dB. And we want to choose d such that

$$P(P_{recd}(d) > -25dBm) = 0.9(1point) \rightarrow$$

$$\begin{aligned} P\left(\frac{P_{recd}(d)-\mu_d}{\sigma_{\psi dB}} > \frac{-25dBm - 30dBm - 10\log_{10}[\frac{1.4\times10^{-3}}{d^2}]}{6}\right) &= 0.9(2points) \\ \Rightarrow \frac{-55dBm - 10\log_{10}[\frac{1.4\times10^{-3}}{d^2}]}{6} &= Q^{-1}(0.9) = 1.2816 \\ \Rightarrow 10\log_{10}[\frac{1.4\times10^{-3}}{d^2}] &= -47.308(1point) \\ \Rightarrow \frac{1.4\times10^{-3}}{d^2} &= 1.86\times10^{-5}(1point) \\ \Rightarrow d &= 8.68m(1point) \end{aligned}$$

8. (2-21) (10points)
   Outage Prob. = Prob. [received $power_{dB} \leqslant Tp_{dB}$]
   Tp = 10dB

   (a) (3points)

$$outageprob. = 1 - Q\left(\frac{Tp - \mu_\psi}{\sigma_\psi}\right) = 1 - Q\left(\frac{-5}{8}\right) = Q\left(\frac{5}{8}\right) = 26\%$$

   (b) (2points)σ_ψ = 4dB, outage prob < 1% ⇒

$$Q\left(\frac{Tp - \mu_\psi}{\sigma_\psi}\right) > 99\% \Rightarrow \frac{Tp - \mu_\psi}{\sigma_\psi} < -2.33 \Rightarrow$$

$$\mu_\psi \geq 19.32dB$$

   (c) (2points)

$$\sigma_\psi = 12dB, \frac{Tp - \mu_\psi}{\sigma_\psi} < -6.99 \Rightarrow \mu_\psi \geq 37.8dB$$

   (d) (3points)For mitigating the effect of shadowing, we can use macroscopic diversity. The idea in macroscopic diversity is to send the message from different base stations to achieve uncorrelated shadowing. In this way the probability of power outage will be less because both base stations are unlikely to experience an outage at the same time, if they are uncorrelated.

9. (2-22) (10points)

$$C = \frac{2}{R^2} \int_{r=0}^{R} rQ\left(a + b\ln\frac{r}{R}\right) dr$$

   To perform integration by parts, we let $du = rdr$ and $v = Q\left(a + b\ln\frac{r}{R}\right)$. Then $u = \frac{1}{2}r^2$ and

$$dv = \frac{\partial}{\partial r}Q\left(a + b\ln\frac{r}{R}\right) = \frac{\partial}{\partial x}Q(x)|_{x=a+b\ln(r/R)}\frac{\partial}{\partial r}\left(a + b\ln\frac{r}{R}\right) = \frac{-1}{\sqrt{2\pi}}\exp(-k^2/2)\frac{b}{r}dr. \quad (1)$$

where $k = a + b \ln \frac{r}{R}$. Then we get

$$C = \frac{2}{R^2}\left[\frac{1}{2}r^2 Q\left(a + b\ln\frac{r}{R}\right)\right]_{r=0}^{R} + \frac{2}{R^2}\int_{r=0}^{R} \frac{1}{2}r^2 \frac{1}{\sqrt{2\pi}} \exp(-k^2/2)\frac{b}{r}dr \quad (2)$$

$$= Q(a) + \frac{1}{R^2}\int_{r=0}^{R} r^2 \frac{1}{\sqrt{2\pi}} \exp(-k^2/2)\frac{b}{r}dr \quad (3)$$

$$= Q(a) + \frac{1}{R^2}\int_{r=0}^{R} \frac{1}{\sqrt{2\pi}} R^2 \exp\left(\frac{2(k-a)}{b}\right) \exp(-k^2/2)\frac{b}{r}dr \quad (4)$$

$$= Q(a) + \int_{k=-\infty}^{a} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-k^2}{2} + \frac{2k}{b} - \frac{2a}{b}\right) dk \quad (5)$$

$$= Q(a) + \exp\left(\frac{-2a}{b} + \frac{2}{b^2}\right)\int_{k=-\infty}^{a} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(k - \frac{2}{b})^2\right) dk \quad (6)$$

$$= Q(a) + \exp\left(\frac{2-2ab}{b^2}\right)\left[1 - Q\left(\frac{a - \frac{2}{b}}{1}\right)\right] \quad (7)$$

$$= Q(a) + \exp\left(\frac{2-2ab}{b^2}\right) Q\left(\frac{2-ab}{b}\right) \quad (8)$$

$$(9)$$

Since $Q(-x) = 1 - Q(x)$.

10. (2-25) (10points)
(Solution provided for the case $a = 0$ (i.e. $\overline{P_r}(r) = P_{min}$), the number will change for the case when $a = -3$ (i.e. $\overline{P_r}(r) = P_{min} + 3$ dB), however the reasoning will remain the same.)

$\gamma/\sigma_{\psi_{\text{dB}}}$	2	4	6
4	0.7728	0.8587	0.8977
8	0.6786	0.7728	0.8255
12	0.6302	0.7170	0.7728

Since $\overline{P_r}(r) \geq P_{min}$ for all $r \leq R$, the probability of non-outage is proportional to $Q\left(\frac{-1}{\sigma}\right)$, and thus decreases as a function of σ. Therefore, C decreases as a function of σ. Since the average power at the boundary of the cell is fixed, C increases with γ, because it forces higher transmit power, hence more received power at $r < R$. Due to these forces, we have minimum coverage when $\gamma = 2$ and $\sigma = 12$. (To this step, 7 points).

By a similar argument, we have maximum coverage when $\gamma = 6$ and $\sigma = 4$. (3 points for this part.) The same can also be seen from this figure:

The value of coverage for middle point of typical values i.e. $\gamma = 4$ and $\sigma = 8$ can be seen from the table or the figure to be 0.7728.

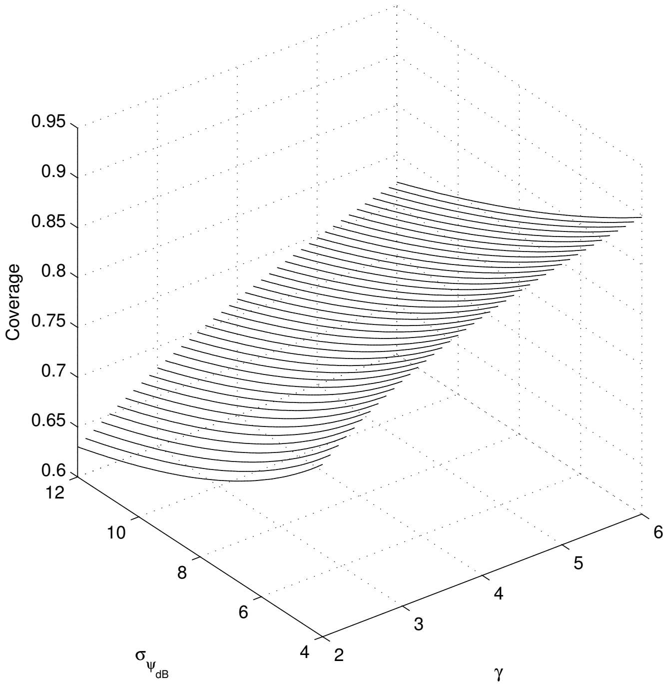

Figure 4: Problem 2-25