EE359, Wireless Communications, Winter 2020
Homework 8 (100 pts)

Due: Saturday (March 14), 11:59 pm

Please refer to the homework page on the website (`ee359.stanford.edu/homework`) for guidelines.
Please note that we have a hard deadline (no late submissions please) this time because we will be uploading solutions soon after.

1. (10 pts) **Multicarrier transmission scheme to mitigate ISI** (Problem 12-4): Consider a data signal with a bandwidth of .5 MHz and a data rate of .5 Mbps. The signal is transmitted over a wireless channel with a delay spread of 10 μs.

   (a) (5 pts) If multicarrier modulation with nonoverlapping subchannels is used to mitigate the effects of ISI, approximately how many subcarriers are needed? What is the data rate and symbol time on each subcarrier? (We do not need to eliminate the ISI completely, so $T_N = T_m$ is sufficient for ISI mitigation.)

   Assume for the remainder of the problem that the nth subcarrier experiences as constant receive SNR(γ_s) of $1000/n$(linear units). The effects of ISI is completely eliminated and fading is approximately constant over a block length.

   (b) (5 pts) Suppose you use adaptive loading (i.e. different constellations on each subcarrier) such that the BER on each subcarrier does not exceed 10^{-3}. Find the MQAM constellation (constellation size is a power of 2) that can be transmitted over each subcarrier while meeting this average BER target using the bound BER $\leq .2e^{-1.5\gamma/(M-1)}$. What is the total data rate of the system with adaptive loading?

2. (10 pts) **On an equivalent OFDM symbol** (Problem 12-10): Show that appending the all-zero prefix to an OFDM symbol and then adding in the tail of the received sequence, as shown in Figure 12.8, results in the same received sequence as with a cyclic prefix. Why might this be more desirable than adding a cyclic prefix?

3. (20 pts) **Constructing circulant matrix from Toeplitz matrix** (Problem 12-11 with two additional questions): Consider a discrete-time FIR channel with

$$h[n] = .7\delta[n] + .5\delta[n-1] + .3\delta[n-3].$$

   Consider an OFDM system with $N = 8$ subchannels. Material in section 12.4.4 will be useful for this question.

   (a) (4 pts) Find the matrix $\mathbf{H}$ corresponding to the matrix representation of the $\mathbf{y} = \mathbf{H}\mathbf{x} + \boldsymbol{\nu}$ given in (12.23).

   (b) (4 pts) Find the circulant convolution matrix $\tilde{\mathbf{H}}$ corresponding to the matrix representation in (12.25), as well as its eigenvalue decomposition $\tilde{\mathbf{H}} = \mathbf{M}\boldsymbol{\Lambda}\mathbf{M}^{\mathbf{H}}$.

   (c) (4 pts) What are the flat fading channel gains associated with each subchannel in the representation of part (b)?

(d) (4 pts) Consider now a point to point link with 2 Tx antennas and 2 Rx antennas. The discrete time FIR channels are as follows:

- From Tx 1 to Rx 1: $h[n] = 0.7\delta[n] + 0.5\delta[n-1] + 0.3\delta[n-3]$
- From Tx 2 to Rx 1: $h[n] = 0.1\delta[n] - 0.7\delta[n-1] + 0.6\delta[n-2]$
- From Tx 1 to Rx 2: $h[n] = 0.8\delta[n] - 0.2\delta[n-1]$
- From Tx 2 to Rx 2: $h[n] = -0.8\delta[n] - 0.1\delta[n-1]$

What are the channel matrices from the transmitters to the receivers (refer to eq (12.23)) ?

(e) (4 pts) How are the eigenvectors of the circulant forms of these matrices related to the **M** above? (*Note that you need not compute the eigendecomposition to answer this. Another hint: the eigenvectors of a general circulant matrix are the columns of a DFT matrix.*)

4. (20 pts) **SIR for a spread signal** (Problem 13-1): In this problem we derive the SIR ratio (13.6) for a randomly spread signal with interference. The correlator output of the ith receiver branch in this system is given by (13.5) as

$$x_i = \int_0^T x(t)s_i(t)dt = \sum_{j=1}^{N}(s_{ij}^2 + I_j s_{ij})$$

(a) (4 pts) Show that the conditional expectation of x_i, conditioned on the fact that $s_i(t)$ is transmitted over the channel, is equal to E_s.

(b) (4 pts) Assuming equiprobable signaling ($p(s_i(t)) = 1/M$), show that $\mathbf{E}[x_i] = E_s/M$.

(c) (4 pts) Show that $\mathrm{Var}[x_i - \sum_{j=1}^N s_{ij}^2 | s_i(t)] = E_s E_J/N$.

(d) (4 pts) Show that (again with equiprobable signaling) $\mathrm{Var}[x_i - \sum_{j=1}^N s_{ij}^2 | s_i(t)]p(s_i(t)) = E_s E_J/NM$.

(e) (4 pts) The SIR is given by

$$\mathrm{SIR} = \frac{\mathbf{E}[x_i]^2}{\mathrm{Var}[x_i - \sum_{j=1}^N s_{ij}^2 | s_i(t)]p(s_i(t))}.$$

Show that

$$\mathrm{SIR} = E_s/E_j \cdot N/M.$$

5. (10 pts) **On RAKE receivers** (Problem 13-13): This problem illustrates the benefits of RAKE receivers and the optimal choice of multipath components for combining when the receiver complexity is limited. Consider a multipath channel with impulse response

$$h(t) = \alpha_0\delta(t) + \alpha_1\delta(t-\tau_1) + \alpha_2\delta(t-\tau_2).$$

The α_i are Rayleigh fading coefficients, but their expected power varies (because of shadowing) such that $E[\alpha_0^2] = 5$ with probability .5 and 10 with probability .5, $E[\alpha_1^2] = 0$ with probability .5 and 20 with probability .5, and $E[\alpha_2^2] = 5$ with probability .75 and 10 with probability .25 (all units are linear). The transmit power and noise power are such that a spread-spectrum receiver locked to the ith multipath component will have an SNR of α_i^2 in the absence of the other multipath components. Assume spreading codes with autocorrelation equal to a delta function are used which implies that there is no interference from other multipath components when a spread-spectrum receiver locks on to a particular multipath component.

(a) (5 pts) What is the outage probability of DPSK modulation at an instantaneous $P_b = 10^{-3}$ for a single-branch spread-spectrum receiver locked to the LOS path?

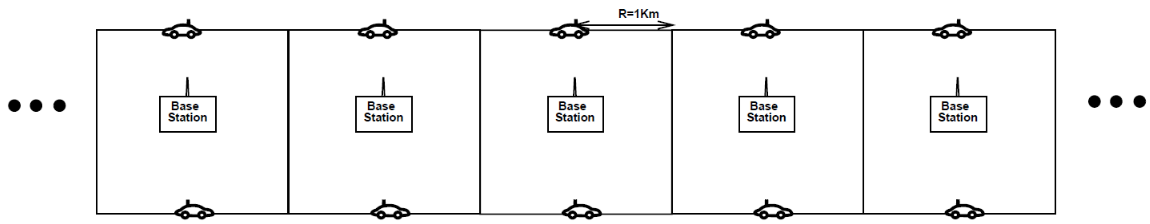

Figure 1: One-dimensional cellular system for Problem 8.

(b) (5 pts) Find the outage probability of DPSK modulation at an instantaneous $P_b = 10^{-3}$ for a three-branch RAKE receiver, where each branch is locked to one of the multipath components and SC is used to combine the paths.

6. (15 pts) **Degradation of WiFi** In a coffee shop, there are M users communicating with a WiFi access point. At every time slot, a user can communicate over any one of N different frequency slots.

   (a) (3 pts) If the access point performs centralized control, it can allocate frequency slots to users at each point in time. What is the expected number of users that are served at any time instant?

   (b) (9 pts) WiFi uses decentralized random access protocols. Here, each user chooses a frequency slot at random. If a frequency slot is picked by a single user, it can be used for communication. Otherwise, there are collisions. Find the expected number of users that are served at any time instant. Evaluate this expression for $N = 10$ and $M = 3, 10$ and 30.

   Hint: Use linearity of expectations

   (c) (3 pts) What fraction of users are served when $M = kN$ for $k \geq 1$ and N is large?

7. (15 pts) **Cellular network** (Problem 15-11) Consider a one-dimensional cellular system deployed along a highway. The system has square cells of length $2R = 2$ km as shown in Fig. 1. This problem focuses on downlink transmission from base stations to mobiles. Assume that each cell has two mobiles located as shown in the figure, so that the mobiles in each cell have the exact same location relative to their respective base stations. Assume a total transmit power at each base station of $P_t = 5$ W, which is evenly divided between the two users in its cell. The total system bandwidth is 100 kHz, and the noise power spectral density at each receiver is 10^{-16} W/Hz. The propagation follows the model $P_r = 100P_t/d^3$. All interference should be treated as AWGN, and interference outside the first ring of interfering cells can be neglected. The system uses a frequency-division strategy, with the bandwidth allocated to each base station evenly divided between the two users in its cell. Neglecting and fading or shadowing, and assuming user rates are equal to the AWGN Shannon capacity of their links, compute the spectral efficiency (sum-rate divided by bandwidth) of each cell when frequencies are reused every other cell (reuse distance 2).