EE359, Wireless Communications, Winter 2020
Homework 3 (100 pts)

Due: Friday January 31 at 4 pm

Please refer to the homework page on the website (`ee359.stanford.edu/homework`) for guidelines.

1. (15 pts) **Channel fading simulator** (Problem 3-12): The goal of this problem is to develop a Rayleigh fading simulator for a mobile communications channel using the method of filtering Gaussian processes that is based on the in-phase and quadrature PSDs described in Section 3.2.1. In this problem you must do the following.

   (a) (10 pts) Develop simulation code to generate a signal with Rayleigh fading amplitude over time. Your sample rate should be at least 1000 samples per second, the average received envelope should be 1, and your simulation should be parameterized by the Doppler frequency f_D . Matlab is the easiest way to generate this simulation, but any code is fine. Write a description of your simulation that clearly explains how your code generates the fading envelope; use a block diagram and any necessary equations. Turn in any code that you use.

   (b) (5 pts) Provide plots of received amplitude (dB) versus time for $f_D = 1, 10,$ and 100 Hz over 2 seconds.

2. (15 pts) **Multipath and narrowband approximation in wireless channels** (Problem 3-17): In this problem we examine the narrowband approximation in the two ray model. Consider the two ray model discussed in Section 2.4 of the text with reflection coefficient $R = -1$, free space path loss for each ray and carrier frequency 2400 MHz. Both the transmitter and the receiver are 40 m high.

   (a) (5 pts) For a separation distance of 20 m, find the impulse response of the equivalent baseband channel in the time domain. What is the delay spread T_m?

   (b) (5 pts) Repeat the computation of the impulse response and delay spread for a separation distance of 2000 m.

   (c) (5 pts) Plot the channel output in the baseband in both the time and frequency domain in each of the above two channels, with $\text{sinc}(ft) \triangleq \frac{\sin(\pi f t)}{\pi f t}$ being the channel input, and $f = 20$ MHz. Is there ISI in either channel? Find the coherence bandwidth of the channel in each case and whether this bandwidth implies flat or frequency selective fading.

3. (20 pts) **Wideband autocorrelation function** (Problem 3-18): Consider a wideband channel characterized by the autocorrelation function

$$A_c(\tau, \Delta t) = \begin{cases} \text{sinc}(W\Delta t) & 0 \leq \tau \leq 10\mu s \\ 0 & \text{else,} \end{cases}$$

   where $W = 100$ Hz and $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$.

   (a) (4 pts) Sketch the scattering function of this channel.

(b) (3 pts) Does this channel correspond to an indoor channel or an outdoor channel, and why?

(c) (5 pts) Compute the channel's average delay spread, rms delay spread, and Doppler spread.

(d) (3 pts) Over approximately what range of data rates will a signal transmitted via this channel exhibit frequency-selective fading?

(e) (5 pts) Assume a system with narrowband binary modulation sent over this channel. Your system has error correction coding that can correct two simultaneous bit errors. Assume also that you always make an error if the received signal power is below its average value and that you never make an error if this power is at or above its average value. If the channel is Rayleigh fading, then what is the maximum data rate that can be sent over this channel with error-free transmission? Make the approximation that the fade duration never exceeds twice its average value.

4. (20 pts) **Scattering function** (Problem 3-19): Let a scattering function $S_c(\tau, \rho)$ be nonzero over $0 <= \tau <= .1$ ms and $-.1 <= \rho <= .1$ Hz. Assume that the power of the scattering function is approximately uniform over the range where it is nonzero.

   (a) (5 pts) What are the multipath spread and the Doppler spread of the channel?

   (b) (5 pts) Suppose you input to this channel two identical sinusoids separated in frequency by Δf. What is the minimum value of Δf for which the channel response to the first sinusoid is approximately independent of the channel response to the second sinusoid?

   (c) (5 pts) For two sinusoidal inputs to the channel $u_1(t) = \sin 2\pi f t$ and $u_2(t) = \sin 2\pi f(t + \Delta t)$, find the minimum value of Δt for which the channel response to $u_1(t)$ is approximately independent of the channel response to $u_2(t)$.

   (d) (5 pts) Will this channel exhibit flat fading or frequency-selective fading for a typical voice channel with a 3-kHz bandwidth? For a cellular channel with a 30-kHz bandwidth?

5. (20 pts) **Power and bandwidth limited regimes in point-to-point AWGN**: Capacity in AWGN is given by $C = B \log_2 \left(1 + \frac{P}{N_0 B}\right)$, where P is the received signal power, B is the signal bandwidth, and $N_0/2$ is the noise PSD (for a white noise process this is constant).

   (a) (10 pts) (Problem 4-1): Find capacity in the limit of infinite bandwidth $B \to \infty$ as a function of P.

   (b) (5 pts) Write the first order approximation of capacity as P becomes small. This is the power limited regime; does doubling bandwidth affect capacity here?

   (c) (5 pts) For a large P, how does the capacity scale with increasing P? This is the bandwidth limited regime; does doubling the bandwidth affect the capacity here?

6. (10 pts) **Successive decoding** (Problem 4-3): Consider two users simultaneously transmitting to a single receiver in an AWGN channel. This is a typical scenario in a cellular system with multiple users sending signals to a base station. Assume the users have equal received power of 10 mW and total noise at the receiver in the bandwidth of interest of 0.1 mW. The channel bandwidth for each user is 20 MHz.

   (a) (5 pts) Suppose that the receiver decodes user 1's signal first. In this decoding, user 2's signal acts as noise (assume it has the same statistics as AWGN). What is the capacity of user 1's channel with this additional interference noise?

   (b) (5 pts) Suppose that, after decoding user 1's signal, the decoder re-encodes it and subtracts it out of the received signal. Now, in the decoding of user 2's signal, there is no interference from user 1's signal. What then is the Shannon capacity of user 2's channel?

   Note: It is shown in Chapter 14 of the first edition of the textbook that the decoding strategy of successively subtracting out decoded signals is optimal for achieving Shannon capacity of a multiuser channel with independent transmitters sending to one receiver.