EE 359 - Wireless Communications - Winter 2008

Homework 4 Solutions (Old)

1. (4-8)

   (a) If neither transmitter nor receiver knows when the interferer is on, they must transmit assuming worst case, i.e. as if the interferer was on all the time,

   $$C = B\log\left(1+\frac{\overline{S}}{N_0B+\overline{I}}\right) = 10.7Kbps.$$

   (b) Suppose we transmit at power S_1 when jammer is off and S_2 when jammer is off,

   $$C = B\max\left[\log\left(1+\frac{S_1}{N_oB}\right)0.75+\log\left(1+\frac{S_2}{N_oB+\overline{I}}\right)0.25\right]$$

   subject to

   $$0.75S_1+0.25S_2=\overline{S}.$$

   This gives $S_1 = 12.25mW$, $S_2 = 3.25mW$ and $C = 53.21Kbps$.

   (c) The jammer should transmit $-x(t)$ to completely cancel off the signal.

2. (4-13)

   (a) C=13.98Mbps

```
MATLAB

Gammabar = [1 .5 .125]; ss = .001; P = 30e-3; N0 = .001e-6;

Bc = 4e6; Pnoise = N0*Bc; hsquare = [ss:ss:10*max(Gammabar)]; gamma
= hsquare*(P/Pnoise);

for i = 1:length(Gammabar)
    pgamma(i,:) = (1/Gammabar(i))*exp(-hsquare/Gammabar(i));
end

gamma0v = [1:.01:2]; for j = 1:length(gamma0v)
    gamma0 = gamma0v(j);
    sumP(j) = 0;
    for i = 1:length(Gammabar)
        a = gamma.*(gamma>gamma0);
        [b,c] = max(a>0);
        gammac = a(find(a));
        pgammac = pgamma(i,c:length(gamma));
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        Pj_by_P = (1/gamma0)-(1./gammac);
        sumP(j) = sumP(j) + sum(Pj_by_P.*pgammac)*ss;
    end
end [b,c] = min(abs((sumP-1))); gamma0ch = gamma0v(c);

C = 0; for i = 1:length(Gammabar)
    a = gamma.*(gamma>gamma0ch);
    [b,c] = max(a>0);
    gammac = a(find(a));
    pgammac = pgamma(i,c:length(gamma));
    C = C + Bc*ss*sum(log2(gammac/gamma0ch).*pgammac);
end
```

(b) C=13.27Mbps

MATLAB

```
Gammabarv = [1 .5 .125]; ss = .001; Pt = 30e-3; N0 = .001e-6;

Bc = 4e6; Pnoise = N0*Bc; P = Pt/3; for k = 1:length(Gammabarv)
    Gammabar = Gammabarv(k);
    hsquare = [ss:ss:10*Gammabar];
    gamma = hsquare*(P/Pnoise);
    pgamma = (1/Gammabar)*exp(-hsquare/Gammabar);
    gamma0v = [.01:.01:1];
    for j = 1:length(gamma0v)
        gamma0 = gamma0v(j);
        a = gamma.*(gamma>gamma0);
        [b,c] = max(a>0);
        gammac = a(find(a));
        pgammac = pgamma(c:length(gamma));
        Pj_by_P = (1/gamma0)-(1./gammac);
        sumP(j) = sum(Pj_by_P.*pgammac)*ss;
    end
    [b,c] = min(abs((sumP-1)));
    gamma0ch = gamma0v(c);
    a = gamma.*(gamma>gamma0ch);
    [b,c] = max(a>0);
    gammac = a(find(a));
    pgammac = pgamma(c:length(gamma));
    C(k) = Bc*ss*sum(log2(gammac/gamma0ch).*pgammac);
end Ctot = sum(C);
```

3. (5-9)

(a) $g(t) = \sqrt{\frac{2}{T}} \quad 0 \leq t \leq T$
$g(T-t) = \sqrt{\frac{2}{T}} \quad 0 \leq t \leq T$
plotted for T=1 , integral value = 2/T = 2

(b) $g(t) = sinc(t) \quad 0 \leq t \leq T$
$g(T-t) = sinc(T-t) \quad 0 \leq t \leq T$
plotted for T=1 , integral value = 0.2470

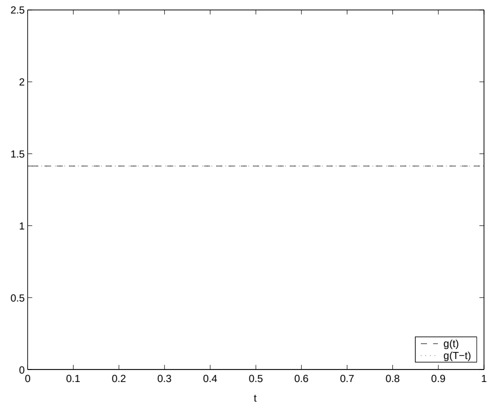

Figure 1: Problem 9a

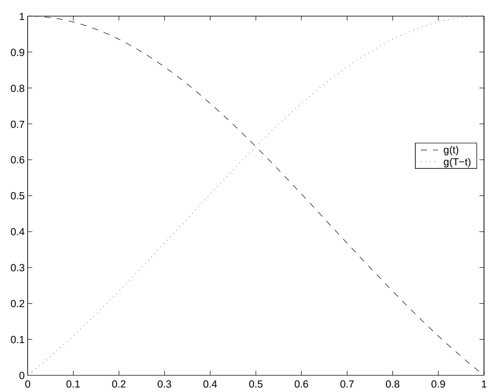

Figure 2: Problem 9b

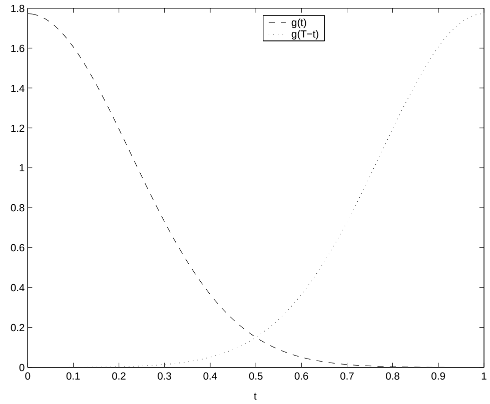

Figure 3: Problem 9c

(c) $g(t) = \frac{\sqrt{\pi}}{\alpha} e^{-\pi^2 t^2/\alpha^2} \quad 0 \leq t \leq T$
$g(T-t) == \frac{\sqrt{\pi}}{\alpha} e^{-\pi^2 (T-t)^2/\alpha^2} \quad 0 \leq t \leq T$
plotted for T=1 , integral value = 0.009

```
MATTLAB CODE T = 1; alpha = 1;

t = [0:.01:T];
%% Part a)
g = repmat(sqrt(2/T),1,length(t)); gm=repmat(sqrt(2/T),1,length(t));
int_a = sum(g.*gm)*.01; plot(t,g,'b--'); hold on; plot(t,gm,'b:');

%% Part b)
figure; g = sinc(t); gm = sinc(T-t); int_b = sum(g.*gm)*.01;
plot(t,g,'b--'); hold on; plot(t,gm,'b:');

%% Part c)
figure; g = (sqrt(pi)/alpha)*exp(-((pi)^2*t.^2)/alpha^2);
gm=(sqrt(pi)/alpha)*exp(-((pi)^2*(T-t).^2)/alpha^2);;
int_c=sum(g.*gm)*.01; plot(t,g,'b--'); hold on; plot(t,gm,'b:');
```

4. (5-11)
(5.40) gives $\frac{1}{4}\sum_{i=1}^{4}\sum_{j=1,j\neq i}^{4} Q\left(\frac{d_{ij}}{\sqrt{2N_0}}\right) = 4.1 \times 10^{-9}$
(5.43) gives $(4-1)Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = 2.3 \times 10^{-8}$
(5.44) gives $\frac{(4-1)}{\sqrt{\pi}} \exp\left(-\frac{d_{min}^2}{4N_0}\right) = 1.9 \times 10^{-7}$
(5.45) gives $M_{d_{min}} Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = 2Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = 1.5 \times 10^{-8}$

```
MATLAB CODE
 Ac = 4;
 s(1,:) = [Ac 0];
 s(2,:) = [0 2*Ac];
 s(3,:) = [0 -2*Ac];
 s(4,:) = [-Ac 0];

sume = 0; for i = 1:4
```

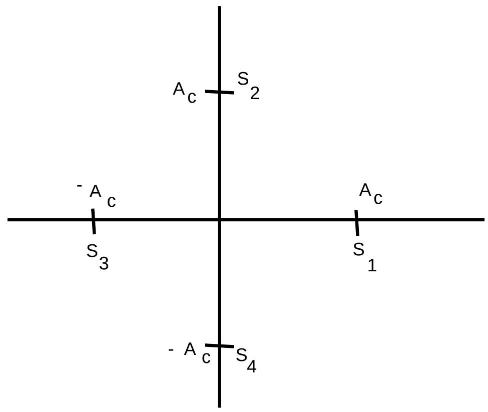

Figure 4: Problem 11

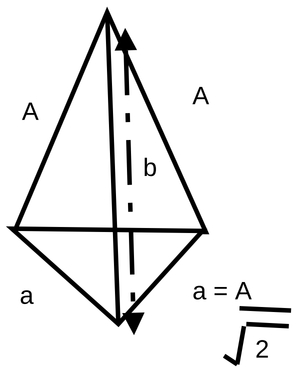

Figure 5: Problem 17a

```
    for j = 1:4
        if j ~= i
            d(i) = norm(s(i,:)-s(j,:));
            sume = sume+Q(d(i)/sqrt(2));
        end
    end
end E1 = .25*sume; dmin = min(d);

E2 = 3*Q(dmin/sqrt(2)); E3 = (3/sqrt(pi))*exp(-dmin^2/4);
E4=2*Q(dmin/sqrt(2));
```

5. (5-17)

   (a) $a = 0.7071A$
   $b = 1.366A$

   (b) $A^2 = r^2\left(2 - 2\cos\frac{\pi}{4}\right)$
   $r = 1.3066A$

   (c) Avg power of 8PSK $= r^2 = 1.7071A^2$
   Avg power of 8 QAM $= 1.1830A^2$
   The 8QAM constellation has a lower average power by a factor of 1.443 (1.593 dB)

   (d) See Fig 6

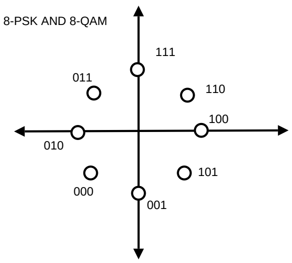

Figure 6: Problem 17d

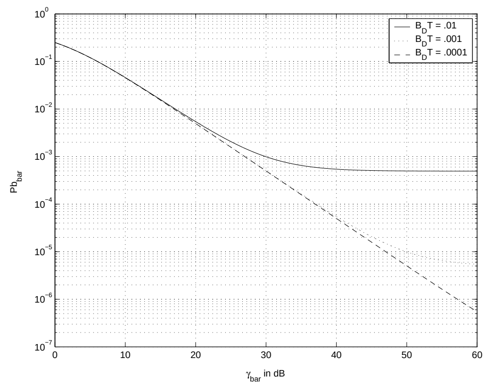

Figure 7: Problem 19

(e) We have 3 bits per symbol $\therefore$ symbol rate $= 30$ Msymbols/sec

6. (5-19)

$$\int_0^T 2\cos(2\pi f_i t)\cos(2\pi f_j t)dt = 0 \Rightarrow \underbrace{\int_0^T 2\cos\left(2\pi(f_i+f_j)t\right)}_{A} + \underbrace{\int_0^T 2\cos\left(2\pi(f_i-f_j)t\right)}_{B}$$

$$A \rightarrow 0 \quad ,\text{as } f_i + f_j \gg 1$$

$$B = \sin\left(2\pi(f_i - f_j)T\right) \quad \text{is 0 first time for } 2\pi(f_i-f_j)T = \pm\pi \Rightarrow |f_i - f_j| = 0.5T$$

```
MATLAB CODE gamma_dB = [0:.01:60]; gamma = 10.^(gamma_dB/10);

vBdT = .01; x = 2*pi*vBdT; rho_C = besselj(0,x);
Pb_bar=.5*((1+gamma*(1-rho_C))./(1+gamma));
semilogy(gamma_dB,Pb_bar);

hold on; vBdT = .001; x = 2*pi*vBdT; rho_C = besselj(0,x);
Pb_bar= .5*((1+gamma*(1-rho_C))./(1+gamma));
semilogy(gamma_dB,Pb_bar,'b:');

vBdT = .0001; x = 2*pi*vBdT; rho_C = besselj(0,x);
Pb_bar=.5*((1+gamma*(1-rho_C))./(1+gamma));
semilogy(gamma_dB,Pb_bar,'b--');
```

7. (6-2)
$p_0 = 0.3, p_1 = 0.7$

(a)

$$
\begin{aligned}
P_e &= Pr(0 \text{ detected, } 1 \text{ sent — } 1 \text{ sent})p(1 \text{ sent}) + Pr(1 \text{ detected, } 0 \text{ sent — } 0 \text{ sent})p(0 \text{ sent}) \\
&= 0.7Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) + 0.3Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)
\end{aligned}
$$

$d_{min} = 2A$

$$
= Q\left(\sqrt{\frac{2A^2}{N_0}}\right)
$$

(b)

$$
p(\hat{m} = 0|m = 1)p(m = 1) = p(\hat{m} = 1|m = 0)p(m = 0)
$$

$$
0.7Q\left(\frac{A + a}{\sqrt{\frac{N_0}{2}}}\right) = 0.3Q\left(\frac{A - a}{\sqrt{\frac{N_0}{2}}}\right) \quad , a > 0
$$

Solving gives us 'a' for a given A and N_0

(c)

$$
p(\hat{m} = 0|m = 1)p(m = 1) = p(\hat{m} = 1|m = 0)p(m = 0)
$$

$$
0.7Q\left(\frac{A}{\sqrt{\frac{N_0}{2}}}\right) = 0.3Q\left(\frac{B}{\sqrt{\frac{N_0}{2}}}\right) \quad , a > 0
$$

Clearly $A > B$, for a given A we can find B

(d) Take $\frac{E_b}{N_0} = \frac{A^2}{N_0} = 10$
In part a) $P_e = 3.87 \times 10^{-6}$
In part b) a=0.0203 $P_e = 3.53 \times 10^{-6}$
In part c) B=0.9587 $P_e = 5.42 \times 10^{-6}$
Clearly part (b) is the best way to decode.

```
MATLAB CODE:
A = 1; NO = .1;
a = [0:.00001:1];
t1 = .7*Q(A/sqrt(NO/2));
t2=.3*Q(a/sqrt(NO/2));
diff = abs(t1-t2);
[c,d] = min(diff);
a(d) c
```