EE 359 - Wireless Communications - Winter 2008

Homework 4 Solutions (New)

1. (4-8)

   (a) If neither transmitter nor receiver knows when the interferer is on, they must transmit assuming worst case, i.e. as if the interferer was on all the time,

   $$C = B \log \left(1 + \frac{\overline{S}}{N_0 B + \overline{I}}\right) = 10.7Kbps.$$

   (b) Suppose we transmit at power S_1 when jammer is off and S_2 when jammer is off,

   $$C = B \max \left[\log\left(1 + \frac{S_1}{N_o B}\right) 0.75 + \log\left(1 + \frac{S_2}{N_o B + \overline{I}}\right) 0.25\right]$$

   subject to

   $$0.75S_1 + 0.25S_2 = \overline{S}.$$

   This gives $S_1 = 12.25mW$, $S_2 = 3.25mW$ and $C = 53.21Kbps$.

   (c) The jammer should transmit $-x(t)$ to completely cancel off the signal.

2. (4-13)

   (a) C=13.98Mbps

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MATLAB

Gammabar = [1 .5 .125]; ss = .001; P = 30e-3; N0 = .001e-6;

Bc = 4e6; Pnoise = N0*Bc; hsquare = [ss:ss:10*max(Gammabar)]; gamma
= hsquare*(P/Pnoise);

for i = 1:length(Gammabar)
    pgamma(i,:) = (1/Gammabar(i))*exp(-hsquare/Gammabar(i));
end

gamma0v = [1:.01:2]; for j = 1:length(gamma0v)
    gamma0 = gamma0v(j);
    sumP(j) = 0;
    for i = 1:length(Gammabar)
        a = gamma.*(gamma>gamma0);
        [b,c] = max(a>0);
        gammac = a(find(a));
        pgammac = pgamma(i,c:length(gamma));
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        Pj_by_P = (1/gamma0)-(1./gammac);
        sumP(j) = sumP(j) + sum(Pj_by_P.*pgammac)*ss;
    end
end [b,c] = min(abs((sumP-1))); gamma0ch = gamma0v(c);

C = 0; for i = 1:length(Gammabar)
    a = gamma.*(gamma>gamma0ch);
    [b,c] = max(a>0);
    gammac = a(find(a));
    pgammac = pgamma(i,c:length(gamma));
    C = C + Bc*ss*sum(log2(gammac/gamma0ch).*pgammac);
end
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(b) C=13.27Mbps

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MATLAB

Gammabarv = [1 .5 .125]; ss = .001; Pt = 30e-3; N0 = .001e-6;

Bc = 4e6; Pnoise = N0*Bc; P = Pt/3; for k = 1:length(Gammabarv)
    Gammabar = Gammabarv(k);
    hsquare = [ss:ss:10*Gammabar];
    gamma = hsquare*(P/Pnoise);
    pgamma = (1/Gammabar)*exp(-hsquare/Gammabar);
    gamma0v = [.01:.01:1];
    for j = 1:length(gamma0v)
        gamma0 = gamma0v(j);
        a = gamma.*(gamma>gamma0);
        [b,c] = max(a>0);
        gammac = a(find(a));
        pgammac = pgamma(c:length(gamma));
        Pj_by_P = (1/gamma0)-(1./gammac);
        sumP(j) = sum(Pj_by_P.*pgammac)*ss;
    end
    [b,c] = min(abs((sumP-1)));
    gamma0ch = gamma0v(c);
    a = gamma.*(gamma>gamma0ch);
    [b,c] = max(a>0);
    gammac = a(find(a));
    pgammac = pgamma(c:length(gamma));
    C(k) = Bc*ss*sum(log2(gammac/gamma0ch).*pgammac);
end Ctot = sum(C);
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3. (6-8)

(a)

$$I_x(a) = \int_0^\infty \frac{e^{-at^2}}{x^2+t^2}dt$$

since the integral converges we can interchange integral and derivative for a¿0

$$\begin{aligned}\frac{\partial I_x(a)}{\partial a} &= \int_0^\infty \frac{-te^{-at^2}}{x^2+t^2}dt \\ x^2 I_x(a) - \frac{\partial I_x(a)}{\partial a} &= \int_0^\infty \frac{(x^2+t^2)e^{-at^2}}{x^2+t^2}dt = \int_0^\infty e^{-at^2}dt = \frac{1}{2}\sqrt{\frac{\pi}{a}}\end{aligned}$$

(b) Let $I_x(a) = y$, we get

$$\begin{aligned}y' - x^2 y &= -\frac{1}{2}\sqrt{\frac{\pi}{a}} \\ \text{comparing with} & \\ y' + P(a)y &= Q(a) \\ P(a) = -x^2 \quad &, \qquad Q(a) = -\frac{1}{2}\sqrt{\frac{\pi}{a}} \\ I.F. = e^{\int P(u)u} = e^{-x^2 a} & \\ \therefore e^{-x^2 a} y &= \int -\frac{1}{2}\sqrt{\frac{\pi}{a}} e^{-x^2 u} du \\ \text{solving we get} & \\ y &= \frac{\pi}{2x} e^{ax^2} erfc(x\sqrt{a})\end{aligned}$$

(c)

$$\begin{aligned}erfc(x\sqrt{a}) &= I_x(a)\frac{2x}{\pi}e^{-ax^2} = \frac{2x}{\pi}e^{-ax^2}\int_0^\infty \frac{e^{-at^2}}{x^2+t^2}dt \\ a = 1 & \\ erfc(x) &= \frac{2x}{\pi}e^{-ax^2}\int_0^\infty \frac{e^{-at^2}}{x^2+t^2}dt \\ &= \frac{2}{\pi}\int_0^{\pi/2} e^{-x^2/sin^2\theta}d\theta \\ Q(x) = \frac{1}{2}erfc(x/\sqrt{2}) &= \frac{1}{\pi}\int_0^{\pi/2} e^{-x^2/2sin^2\theta}d\theta\end{aligned}$$

4. (6-10)
$T_s = 15\mu sec$
at 1mph $T_c = \frac{1}{B_d} = \frac{1}{v/\lambda} = 0.74s \gg T_s$
$\therefore$ outage probability is a good measure.
at 10 mph $T_c = 0.074s \gg T_s$ $\therefore$ outage probability is a good measure.
at 100 mph $T_c = 0.0074s = 7400\mu s > 15\mu s$ outage or outage combined with average prob of error can be a good measure.

5. (6-12)

(a) When there is path loss alone, $d = \sqrt{100^2 + 500^2} = 100\sqrt{6} \times 10^3$
$P_e = \frac{1}{2}e^{-\gamma_b} \Rightarrow \gamma_b = 13.1224$
$\frac{P_\gamma}{N_0 B} = 13.1224 \Rightarrow P_\gamma = 1.3122 \times 10^{-14}$
$\frac{P_\gamma}{P_t} = \left[\frac{\sqrt{G}\lambda}{4\pi d}\right]^2 \Rightarrow 4.8488W$

(b)

$$x = 1.3122 \times 10^{-14} = -138.82dB$$
$$P_{\gamma,dB} \sim N(\mu P_\gamma, 8), \sigma_{dB} = 8$$

$$\begin{aligned} P(P_{\gamma,dB} \geq x) &= 0.9 \\ P\left(\frac{P_{\gamma,dB} - \mu P_\gamma}{8} \geq \frac{x - \mu P_\gamma}{8}\right) &= 0.9 \\ \Rightarrow Q\left(\frac{x - \mu P_\gamma}{8}\right) &= 0.9 \\ \Rightarrow \frac{x - \mu_{P_\gamma}}{8} &= -1.2816 \\ \Rightarrow \mu_{P_\gamma} = -128.5672dB = 1.39 \times 10^{-13} \\ P_t = 51.36W \end{aligned}$$

6. (6-16)
   For DPSK in Rayleigh fading, $\overline{P_b} = \frac{1}{2\overline{\gamma_b}} \Rightarrow \overline{\gamma_b} = 500$
   $N_oB = 3 \times 10^{-12} mW \Rightarrow P_{target} = \overline{\gamma}_b N_0 B = 1.5 \times 10^{-9} mW$ = -88.24 dBm

   Now, consider shadowing:
   $P_{out} = P[P_r < P_{target}] = P[\Psi < P_{target} - \overline{P_r}] = \Phi\left(\frac{P_{target} - \overline{P_r}}{\sigma}\right)$
   $\Rightarrow \Phi^{-1}(.01) = 2.327 = \frac{P_{target} - \overline{P_r}}{\sigma}$
   $\overline{P_r} = -74.28$ dBm $= 3.73 \times 10^{-8}$ mW $= P_t\left(\frac{\lambda}{4\pi d}\right)^2$
   $\Rightarrow$ d = 1372.4 m

7. (6-17)

   (a) From simplified path-loss model:

$$\begin{aligned} P_r &= P_t k (\frac{d_0}{d})^\gamma \\ &= 10^{-7} = -40dBm \end{aligned}$$

   (b)

$$\bar{P}_b = \frac{1}{4\bar{\gamma}_b}$$
$$\Rightarrow \bar{\gamma}_b \geq \frac{1}{4\bar{P}_b} = 2.5 \times 10^3$$

$$\bar{p}_{min} = \bar{\gamma}_b N_0 B = 7.5 \times 10^{-7} = -31.25dBm$$

   (c)

$$c = Q(a) + \exp\frac{2 - 2ab}{b^2} Q\left(\frac{2 - ab}{b}\right) = 31\%$$

   where

$$a = \frac{\bar{P}_{min} - \bar{P}_r(R)}{\sigma_{\psi dB}}$$
$$b = \frac{10\gamma \log e}{\sigma_{\psi dB}}$$