EE 359 MIDTERM - WINTER 2020

The exam is open notes and open book. You may use a computer offline (to read notes saved on your device). You may also use a calculator (which may be your phone or computer). The use of MATLAB (or similar software) is not allowed. Communicating with others during the exam is not allows.

1. **Trump's Outage at Mar-a-Lago (30 points)**
   Donald Trump is driving his golf cart while golfing at his resort at Mar-a-Lago. On his drive, his channel impulse response is $\alpha_1(t)\delta(\tau)+\alpha_2(t)\delta(\tau-.2\mu\text{sec})+\alpha_3(t)\delta(\tau-.67\mu\text{sec})$, where the $\alpha_i(t)$ are i.i.d. complex Gaussian processes with independent real and imaginary parts of equal power. The receiver uses standard demodulation with no ISI compensation.

   (a) (4 pts) What fading distribution does the envelope of the channel impulse response follow?

   (b) (6 pts) Suppose the signal $s(t)=\sin(2\pi ft)+\cos[2\pi(f+\Delta f)t]$ is the channel input. Approximate the minimum value of Δf so that the channel output is given by $r(t)=c_1(t)\sin(2\pi ft)+c_2(t)\cos[2\pi(f+\Delta f)t]$ and the complex constants $c_1(t)$ and $c_2(t)$ are approximately independent.

   (c) (10 pts) Assume a BPSK modulated signal is transmitted over this channel, and its average received SNR is 25 dB. For a data rate of $R=10$ Kbps, what is the average probability of error? Trump realizes 10Kbps is too low a data rate for him to effectively communicate with William Barr while he is on the course, so he increases his data rate to $R=100$ Kbps. What is the probability of error now?

   (d) (10 pts) Assume now that the $\alpha_i(t)$ has a combination of fast Rayleigh fading and slow shadowing. The shadowing is uniformly distributed over [50, 2500] linear units. Trump is transmitting a BPSK signal with a data rate of $R=10$ Kbps. Suppose that an outage occurs when the average probability of error due to Rayleigh fading exceeds 10^{-3}. What is the outage probability?

2. **The Nevada Caucuses at Full Capacity (35 points)**

   The Nevada Democrats are preparing for the Nevada caucus. Tom Perez, the chairman of the Democratic National Committee, is traveling between three caucus locations to ensure that the results can be promptly reported and avoid a repeat of issues in the Iowa caucus. Assuming a fixed transmit power $\overline{P}$ and a channel bandwidth of 15 MHz, Perez's received SNR associated with each channel state is $\gamma_1=0$dB, $\gamma_2=6$dB, and $\gamma_3=13$dB, respectively. The probabilities associated with each state (i.e. each caucus location) are $p(\gamma_1)=.2$, $p(\gamma_2)=.5$, $p(\gamma_3)=.3$. Tom Perez's cell phone was a high-tech present from Michael Bloomberg, so both its transmitter and receiver have perfect instantaneous estimates of the channel.

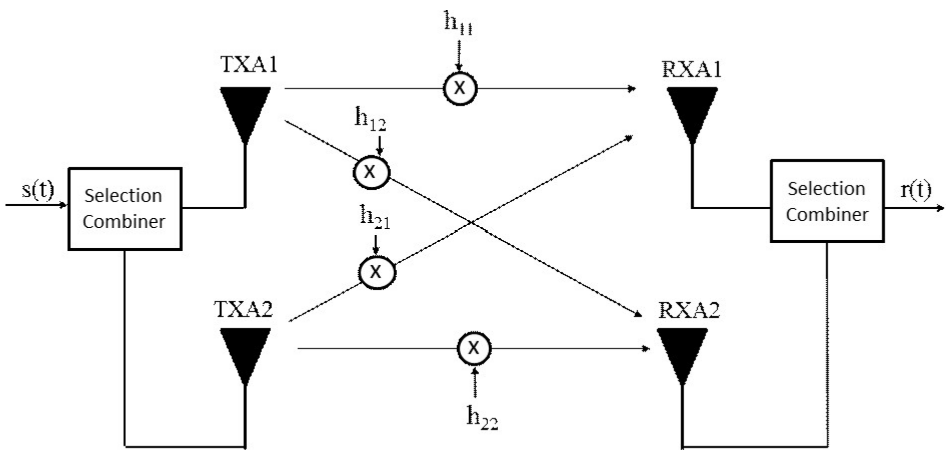
   (a) (15 pts) Find the optimal adaptive transmission strategy and associated Shannon capacity (in bps) of Tom Perez's channel assuming perfect instantaneous transmitter and receiver channel knowledge.

   (b) (7 pts) Consider now a truncated channel inversion policy. Find the truncated channel inversion adaptive power policy that maximizes Tom Perez's outage capacity and find this maximum rate (in bps) and the associated outage probability.

   (c) (3 pts) In order to ensure that he can make reports in real-time without his call being dropped, Tom Perez switches the phone to minimum outage mode. Find the truncated channel inversion adaptive power policy that minimizes outage probability and find the associated outage capacity (in bps) and outage.

   (d) (10 pts) Michael Bloomberg buys the DNC more spectrum in an effort to win over delegates. So the channel bandwidth is increased to 30 MHz but the transmit power must remain fixed at the constant value $\overline{P}$ due to wideband power amplifier limitations. Find the Shannon capacity in this case. Is it better for capacity to adapt power or use double the bandwidth? The answer is why the spectrum cost Bloomberg billions.

3. **Anti-lopsided Diversity aka MIMO (35 Points)**
You are working for Diversity, Inc., a company that specializes in building wireless systems with transmitter or receiver diversity. One fine day, you have a **brilliant idea** - why not put antennas arrays at both the transmitter and receiver rather than only at one end or the other (unfortunately, this was discovered by A. Paulraj in the 1980s and ignited the field of MIMO, but you don't know this and neither does your boss). In order to evaluate the benefits of your idea, you decide to analyze it for a simple 2x2 system, with two transmit antennas and two receive antennas. The channel between transmit antenna i and receive antenna j has complex channel gain h_{ij}, i.e. h_{ij} is the gain from the ith transmit antenna (TXAi) to the jth receive antenna (RXAj). For example, h_{11} defines the channel gain from transmit antenna 1 (TXA1) to receive antenna 1 (RXA1). Assume the h_{ij} are i.i.d. with $|h_{ij}|$ Rayleigh-distributed, and that for a transmit power P_t and noise power N_0B, $\overline{\gamma} = E[P_t|h_{ij}|^2/(N_0B)] = 15$ dB. We assume perfect channel knowledge, i.e. perfect knowledge of all coefficients $\{h_{ij}\}$ at the transmitter and receiver. To keep things simple, you design this 2x2 MIMO system with selection combining (SC) at the transmitter and receiver, so that at each symbol time, the transmitter and receiver select the combination of transmit antenna TXAi and receiver antenna RXAj that maximizes the output SNR of the received signal $r(t)$. This 2x2 SC-MIMO system is shown in the figure below.



(a) (8 pts) Given instantaneous channel gains h_{11}, h_{12}, h_{21}, and h_{22}, which transmit antenna TXAi and receive antenna RXAj should be selected to maximize the instantaneous SNR of the received signal $r(t)$.

(b) (10 pts) Derive the PDF of the instantaneous SNR of the received signal. What is the expected value of the received SNR?

(c) (10 pts) What is the probability that a BPSK signal sent over this SC-MIMO system has $P_b > 10^{-6}$? Use the approximation that $Q(x) \approx 0.5e^{-x^2/2}$.

(d) (7 pts) Suppose we used a lopsided diversity system with the same number of antennas and combining strategy. That is, we use a system with a single transmit antenna and 3 receive antennas under SC. Assume each branch has i.i.d fading with an average SNR of 15dB, like before. What is the probability that a BPSK signal sent over this system has $P_b > 10^{-6}$? Compare with your answer in part (c) - how much is gained in terms of outage probability by balancing the number of antennas on each side? Based on your answer, you run to tell your boss who immediately doubles your salary.

This problem shows that balancing the number of antennas between the transmitter and receiver provides significant diversity gain relative to a lopsided system. We will see the performance gains of a system with balanced antennas apply to both diversity and capacity when we study MIMO systems.