# EE 359 - Wireless Communications - Autumn 2015

## Final: Total 200 Points, Duration: 3:30pm-6:30pm

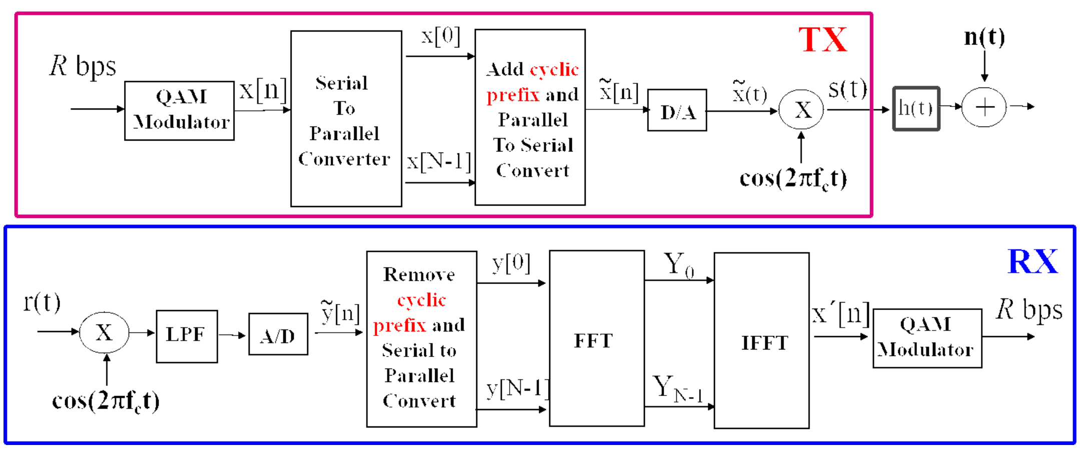
*The exam is open book and notes. Please state all assumptions used in your calculations. You may use any derivations or statements from the book as long as you cite where they come from.*

1. **Problem 1 (50 points): Short Answer**

   (a) Consider an adaptive system with instantaneous SNR at the transmitter $\hat{\gamma}$ that is not necessarily equal to the true SNR $\gamma$. Assume $\hat{\gamma}$ has the same distribution as $\gamma$, so $p(\hat{\gamma}) = p(\gamma)$. For the optimal variable-rate variable-power MQAM scheme with no restrictions on rate or power, suppose that the transmit power and rate is adapted relative to $\hat{\gamma}$ instead of $\gamma$. Will the average transmitted data rate and average transmit power be larger, smaller, or the same as under perfect channel estimates ($\hat{\gamma} = \gamma$), and why? If over a given symbol time $\hat{\gamma} > \gamma$, will the probability of error associated with that symbol be larger or smaller than the target value and why?

   (b) Consider a 2x2 MIMO system with SNRs $\gamma_{ij}$ from the $i$th transmit antenna to the $j$the receiver antenna. What is the SNR of a diversity system with transmit selection combining and receiver MRC? Compare to the SNR of a diversity system with MRC transmit diversity and selection combining in the receiver. In both cases assume perfect channel knowledge on both ends.

   (c) Consider an OFDM system with data rate $R$ and transmitted signal bandwidth $R = B >> B_c$, where $B_c$ is the coherence bandwidth of the channel on which it operates. In the uplink (mobile to base station) LTE systems the IFFT at the transmitter is sometimes moved to the receiver, as shown in the figure below, to save processing power in the mobile. The cyclic prefix is the same as in a standard OFDM system, with length $\mu$ equal to the channel delay spread over a symbol time. Will the transmitted signal $s(t)$ experience flat or frequency selective fading and why? Can the effects of this fading be removed by inserting a block between the FFT and IFFT and, if so, how? Is the data rate of this system larger, smaller, or the same as standard OFDM (with IFFT at the transmitter) and why?




   (d) **Performance of RAKE Receivers:** Consider a 2-branch RAKE receiver. The fading distribution on each branch is $p(\gamma)$ where $\int_0^\infty p(\gamma)e^{-x\gamma}d\gamma = .01\overline{\gamma}/\sqrt{x}$. Find the average $P_b$ for a spread-spectrum BPSK signal where the RAKE receiver has 2-branch diversity with MRC combining, and each branch has an average SNR of 10 dB and experiences independent fading with distribution $p(\gamma)$. (Note: $\sin^2 x = .5(1 - \cos(2x))$. You can neglect the fact that the pdf of $p(\gamma)$ does not integrate to one.

2. **Problem 2 (50 points): Hillary Clinton, Spread Spectrum, and RAKE Receivers**

This problem illustrates the benefits of RAKE receivers in multipath channels with *both* fast fading and shadowing. Hillary Clinton is driving to a campaign rally in Iowa. Her private line iPhone to Bill has a multipath channel with impulse response

$$h(t) = \alpha_0\delta(t - \tau_0) + \alpha_1\delta(t - \tau_1).$$

The $\alpha_i$ are Rayleigh fading coefficients, but their expected power varies due to shadowing by grazing cows such that $E[\alpha_0^2] = 15$ with probability .35 and 20 with probability .65, $E[\alpha_1^2] = 5$ with probability .2 and 25 with probability .8 (all units are linear). The transmit power and noise power are such that Hillary's spread spectrum receiver locked to the $i$th multipath component will have an SNR of $\alpha_i^2$ in the absence of the other multipath components. Assume maximal linear spreading codes that repeat every $20T_c$ and that a timing offset of $.1T_c$ at the receiver (i.e. the spreading code in branch $i$ of the receiver is $s_c(t - \tau_i - .1T_c)$). Unless stated otherwise, you can neglect the effects of ISI between paths.

(a) Find Hillary's outage probability (probability that the instantaneous BER exceeds the target BER) for a cheap phone with a single branch spread spectrum receiver where the receiver locks to the branch that minimizes the outage probability of DPSK modulation for a $10^{-3}$ BER target.

(b) Now assume Hillary paid more for her iPhone to get a 2-branch RAKE receiver with selection combining, where each branch is locked to one of the multipath components. Find the outage probability for DPSK modulation with a target BER of $10^{-3}$. By how much does the RAKE receiver reduce outage probability compared to your answer in part (a)?

(c) For Hillary's cheap phone with a single-branch spread spectrum receiver, which branch should the receiver lock to minimize the average probability of error under DPSK modulation. Find this average probability of error.

(d) No longer ignoring the effect of ISI between paths, let $\tau_1 = \tau_0 + 19.7T_c$. Assuming the ISI is treated as noise with power equal to the average power of the ISI, find the distribution of SINR on each branch of Hillary's expensive phone with a two-branch RAKE receiver with each branch locked to one of the multipath components with timing offset $.1T_c$. For this distribution, assuming selection combining of the branches, find the outage probability for DPSK modulation with a target BER of $10^{-3}$. How much does ISI increase outage compared to your answer in part (b)?

3. **Problem 3 (50 Points): Adele and OFDM Systems**

Adele is on her way to her next concert. She is trying to download the latest sales figures for her new album. Her wireless channel has a power delay profile $A_c(\tau) = 10^5 e^{-10^5\tau}, \tau > 0$, where $\tau$ is in seconds. Assume the distribution of the received signal power is exponential.

(a) What is the average delay spread, rms delay spread, and coherence bandwidth of Adele's channel?

(b) Assuming the OFDM system has a 400KHz bandwidth, find the required number of OFDM subchannels to ensure that each subchannel experiences fading that is approximately flat, assuming this number is a power of two for implementation reasons. Also find the length of the cyclic prefix such that there is approximately zero ISI between OFDM blocks.

(c) For the subchannels in part (b), suppose power is allocated based on channel inversion across all subchannels and with this power allocation each subchannel has an SNR of 16 dB. What is Adele's channel capacity?

(d) Again assuming the channel inversion of part (c) resulting in subchannel SNR of 16 dB, what data rate can Adele achieve if the OFDM system is restricted to use constellations in each subchannel that are restricted to BPSK or MQAM of constellation size equal to a power of two, with a target BER of $10^{-4}$. You can use the BER bound $P_b \leq .2e^{-1.5\gamma/(M-1)}$ in your calculations and assume a symbol time per subchannel $T_c = 1/B_c$.

4. **Problem 4 (50 points): MIMO Dreaming of Tahiti**

You have a new startup building MIMO systems, which you are trying to optimize for complexity versus performance. You test your system over a MIMO channel with the following channel matrix

$$\mathbf{H} = \begin{bmatrix} .7 & .5 & .1 \\ .4 & .3 & .7 \\ .8 & .3 & .6 \end{bmatrix} = \begin{bmatrix} .5080 & -.7803 & -.3648 \\ .5254 & .6163 & -.5867 \\ .6826 & .1063 & .7230 \end{bmatrix} \begin{bmatrix} 1.5172 & 0 & 0 \\ 0 & 0.5002 & 0 \\ 0 & 0 & 0.1674 \end{bmatrix} \begin{bmatrix} .7328 & .4063 & .5458 \\ -.4291 & -.3466 & .8341 \\ .5281 & -.8455 & -.0797 \end{bmatrix}$$

Assume the total transmit power is 10 dBm, the noise power at each receive antenna is 0 dBm, and the system bandwidth is $B = 1$ MHz.

(a) This system has three spatial dimensions: what is the SNR $\gamma_i$ of each dimension assuming the full transmit power is allocated to it?

(b) Find the transmit precoding and receiver shaping matrices used in the transmitter and receiver under spatial multiplexing (all 3 spatial dimensions used), beamforming (one spatial dimension used), and a combination of spatial multiplexing and beamforming (two spatial dimensions used).

(c) You first consider the capacity tradeoffs associated with using different spatial dimensions. Find the optimal power and rate adaptation and the associated MIMO channel capacity of this MIMO system under spatial multiplexing versus beamforming. How many dimensions are used to achieve capacity?

(d) Find the maximum data rate that can be sent under spatial multiplexing with optimal and with equal power allocation over all three channels and compare with the maximum data rate obtained via beamforming. Assume a target BER of $P_b \leq 10^{-3}$ with unrestricted modulation (i.e. $M$ can take any value). Use the BER bound $P_b \leq .2e^{-1.5\gamma/(M-1)}$ for your calculations and assume a symbol time $T = 1/B$. Based on your calculations you recommend to your CEO to go with the MIMO system that has the best performance as the cost of complexity will decrease with time. With this design your company gets lots of business as the performance-leader and goes IPO; you make it rich and move to Tahiti all based on your EE359 knowledge.