

Quiz questions for §1, *Matrix terminology and notation*.

True or false?

1. By convention, we consider  $[0.1 \ 0.3]$  and  $\begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}$  to be the same.
2. The matrix  $\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -2 & 1 \end{bmatrix}$  has dimensions  $2 \times 3$ .
3. The matrices  $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$  and  $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$  are equal, since they differ in the last column only, which is zero.
4. The 2, 1 entry of  $\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -2 & 1 \end{bmatrix}$  is 0.
5. The matrix  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$  is an identity matrix.
6. Suppose we know that  $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = e_2$ , the second unit vector (or standard basis vector).  
Then we can conclude  $a = 0$ .

Quiz questions for §2, *Matrix operations*.

1. The transpose of the matrix  $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$  is
  - (a)  $\begin{bmatrix} 2 & 1 \\ 4 & 3 \\ 6 & 5 \end{bmatrix}$
  - (b)  $\begin{bmatrix} 5 & 6 \\ 3 & 4 \\ 1 & 2 \end{bmatrix}$
  - (c)  $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$
  - (d)  $\begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix}$

- (e) not defined
- (f) none of the above

2.  $[1 \ 2 \ 3]^T + \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$  is

(a)  $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$

(b)  $\begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$

(c)  $[5 \ 7 \ 9]$

(d) not defined

(e) none of the above

3. If  $Q = [1 \ 2 \ 0]$  and  $R = \begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix}$ , then the product  $RQ$  is

(a) the scalar (or  $1 \times 1$  matrix)  $-2$

(b) the matrix  $\begin{bmatrix} -2 & -4 & 0 \\ -1 & -2 & 0 \\ 3 & 6 & 0 \end{bmatrix}$

(c) not defined

(d) none of the above

4. If  $A$  and  $B$  are  $3 \times 3$  matrices, then  $AB \neq BA$ .

**False.** We usually do not have  $AB = BA$ , but there are cases when we do have  $AB = BA$ . One example: take  $A = B = I$ .

5. Let  $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ . Then:

(a)  $A$  is singular

(b)  $A^{-1} = A^T$

- (c)  $A^{-1} = -A$
- (d)  $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$
- (e) none of the above

6. Suppose  $A$  and  $B$  are  $5 \times 5$  matrices. What is  $(A + B)^2 - (A - B)^2$ ?

- (a) this expression doesn't make sense
- (b)  $4AB$
- (c)  $2AB + 2BA$
- (d) none of the above

Quiz questions for §3, *Linear equations and matrices*.

1. Which of the following are linear functions? Here  $y = f(x)$ , where  $x$  is a 2-vector, and  $y$  is a 2-vector.

- I.  $y_1 = (x_1 - x_2)/2, y_2 = (x_1 + x_2)/2$ .
- II.  $y_1 = 1 - x_1 + x_2, y_2 = x_1$ .

- (a) Both I and II.
- (b) I but not II.
- (c) II but not I.
- (d) Neither is linear.

2. Which of the following are linear functions? Here  $y = f(x)$ , where  $x$  is a 3-vector, and  $y$  is a 2-vector.

- I.  $y_1 = 3x_3 - x_2, y_2 = x_2 + 2x_3 - x_1^2$ .
- II.  $y_1 = x_2, y_2 = 0$ .

- (a) Both I and II.
- (b) I but not II.
- (c) II but not I.
- (d) Neither is linear.

3. The set of linear equations

$$x_1 + x_2 = x_3 - x_1, \quad 3 - x_2 = 2x_1$$

can be represented as  $Ax = b$ , for which of the following  $A, b$ ?

I.  $A = \begin{bmatrix} 2 & 1 & 0 \\ 2 & 1 & -1 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 0 \end{bmatrix}.$

II.  $A = \begin{bmatrix} 2 & 1 & -1 \\ -2 & -1 & 0 \end{bmatrix}, b = \begin{bmatrix} 0 \\ -3 \end{bmatrix}.$

- (a) Both I and II.  
 (b) I but not II.  
 (c) II but not I.  
 (d) Neither.
4. Which of the following matrix expressions gives the values of three numbers  $x_1, x_2,$  and  $x_3$ , that satisfy the following:
- The average of the first two numbers is 1.
  - The average of the last two numbers is 3.
  - The average of all three numbers is 2.

We define  $A = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}.$

- (a)  $x = bA^{-1}$   
 (b)  $x = A^{-1}b$   
 (c)  $x = Ab$   
 (d)  $x = bA$   
 (e)  $x = A^T b$   
 (f)  $x = b^T A$
5. Solving a set of 100 linear equations in 100 variables
- (a) is easily done by hand  
 (b) is a dangerous task that should be done by certified professionals only  
 (c) can be done in less than a second on a small PC  
 (d) is a problem that rarely comes up in practical situations  
 (e) requires about a minute on a professional workstation computer
6. If the 5-vector  $x$  is the solution of the linear equations  $Ax = b$ , where  $A$  is a  $5 \times 5$  nonsingular matrix, and  $y$  is the solution of the linear equations  $Ay = 2b$ , what can you say about  $x$  and  $y$ ?

(a)  $y = x/2$

(b)  $y = 2x$

(c) nothing, because the relation between  $x$  and  $y$  is complicated

(d)  $x = y$