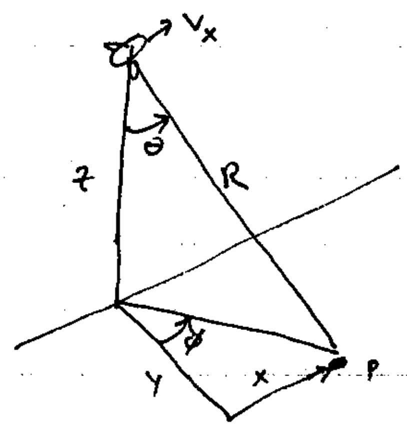
EE 262

Doppler history of a point target

Let's now consider how the Doppler frequency of a point at arbitrary location within our antenna beam:



We have seen the relationship between velocity and Doppler frequency

$$f_d = \frac{2\, v \cdot u}{\lambda}$$

How do we express this in terms of our geometry above? The vector from the radar to the object R is

$$R = (x, y, z)$$

and the velocity is assumed in the x-direction only

$$v = (v_x, 0, 0)$$

Hence

$$f_d = \frac{2}{\lambda}\,(v_x, 0, 0) \cdot \frac{(x, y, z)}{|R|}$$

$$= \frac{2}{\lambda}\, v_x \cdot \frac{x}{|R|}$$

or, since $r = |R|$,

$$f_d = \frac{2}{\lambda}\, v_x \cdot \frac{x}{r}$$

While a simple relation, we'd like to relate f_d to our look and squint angles. Recalling from previous work

$$\sin\theta = \frac{\sqrt{x^2+y^2}}{r}$$

and

$$\sin\phi = \frac{x}{\sqrt{x^2+y^2}}$$

we obtain

$$f_d = \frac{2v_x}{\lambda}\sin\theta\sin\phi$$

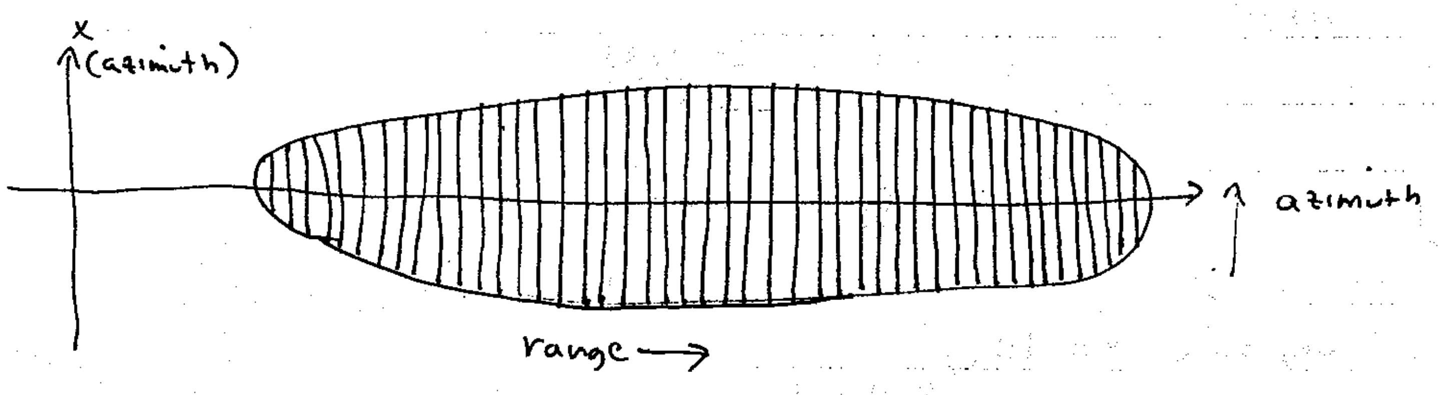
If we assume all motion is in the x direction and let $v = v_x$

$$\boxed{f_d = \frac{2v}{\lambda}\sin\theta\sin\phi}$$

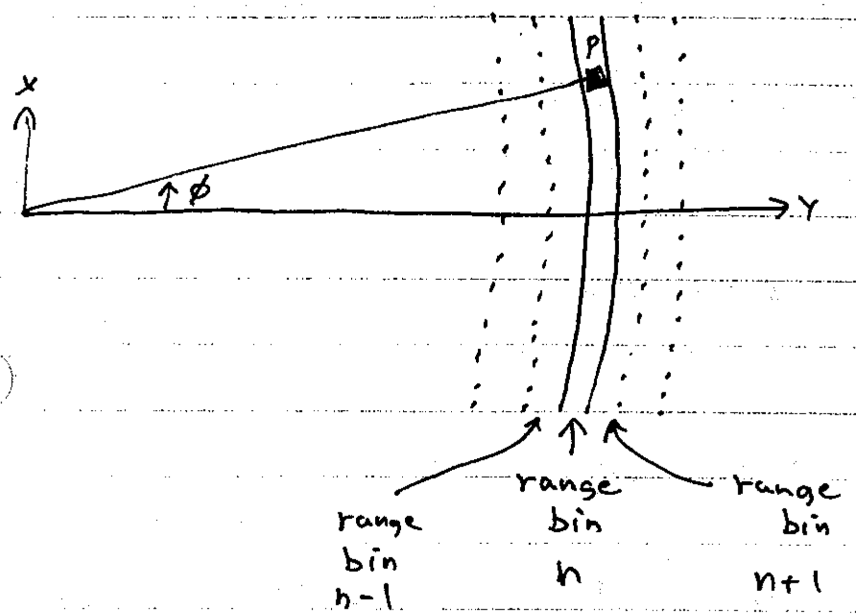
This is the usual Doppler equation in terms of imaging angles. Note that we can also use the total squint angle sq as in

$$f_d = \frac{2v}{\lambda}\sin(sq)$$

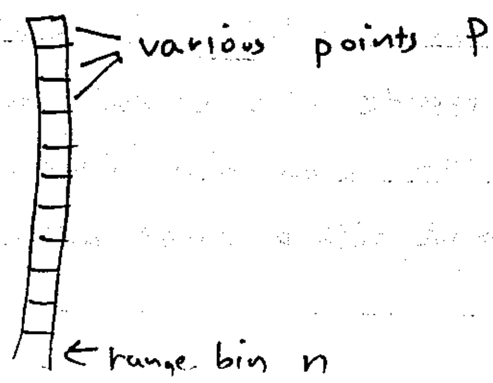
What does this imply for imaging? Consider the following picture of the illuminated part of the radar swath at one instant in time:



Each vertical "stripe" represents one range resolution element, or "bin", obtained by processing the echo in range. As discussed previously, all scatterers within a range bin for the full extent of the azimuth beamwidth contribute to the return. Let's blow up the image of part of the ~~bin~~ pattern, highlighting the return from one bin:



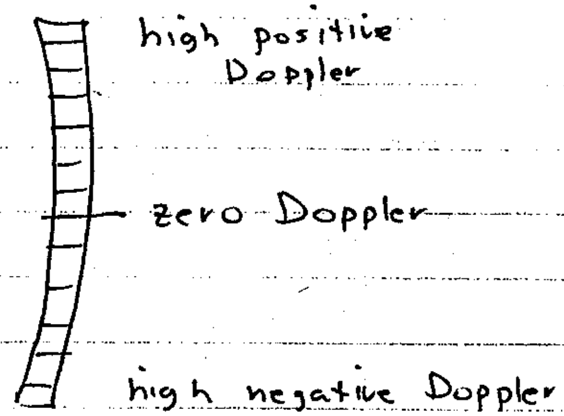
The energy from bin n, which is located at constant range r, and consequently constant look angle θ, from the radar, consists of energy from scatterers at many positions P in azimuth. If we could distinguish among these, we could improve our resolution in the azimuth direction:



But, we can distinguish these. We know that

$$f_d = \frac{2v}{\lambda} \sin\theta \sin\phi$$

hence by decomposing the echo from a given range bin into separate frequency components, we can solve for the squint angle and hence the position of the various scatterers:



This method of deriving resolution in azimuth is called range-Doppler mapping. It is also equal to a technique that is called unfocused SAR processing.

Implementation of Unfocused SAR algorithm

Since we want to sort the energy from each range bin by frequency, a simple way to generate the unfocused SAR image is simply to calculate the Fourier transform of each range bin. This provides the needed mapping.

Note, however, that in our above discussion we have considered illuminating the ground with a single pulse of radar energy. It therefore results in only measurement at each range bin. We can't very well decompose one number into a series of Doppler values. We will therefore operate the radar in a different manner: we will transmit a series of pulses, range compress each to obtain a series of range bin measurements, and transform this series on a bin by bin basis to create the unfocused image.

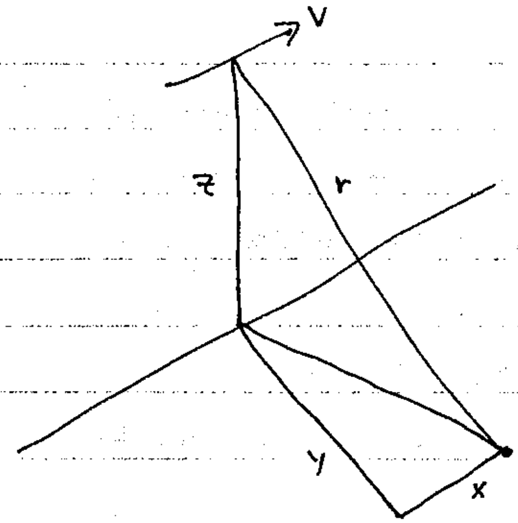
Relation between phase (or range)-change and Doppler

You may be uncomfortable with the notion of measuring Doppler frequency by multiple pulses because the platform is moving during the pulsing period, therefore not exactly the same area is illuminated by each pulse.

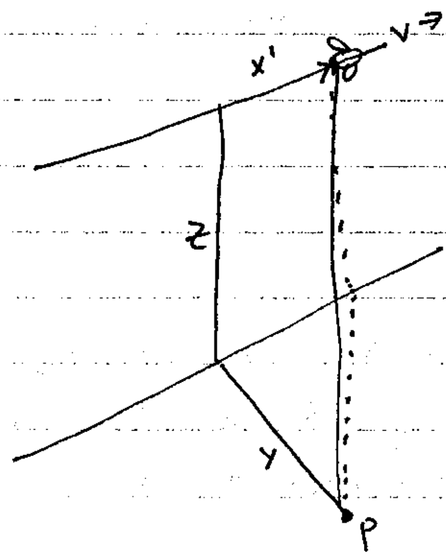
This in fact is central to how our imaging algorithms work, so let's look in more detail as to how the echo from a given scatterer varies in time as the radar flies by.

Variation of range with time. We begin by deriving the variation of the range to an object as a function of time. We know that we can describe the range to an object at one time as

$$r = \sqrt{x^2+y^2+z^2}$$



But, the platform itself is in motion. Since the motion is along the x-direction, the value of x in the above equation is itself a function of time. Let's therefore define a coordinate system fixed on the object and examine the range as the airplane flies by:



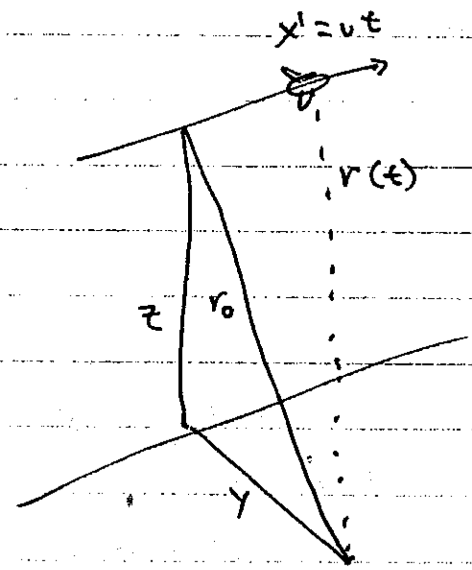
The airplane is located at position x' in this system. Defining as time 0 the instant in which the platform flies closest to the point P, we can express the location of the plane x' as

$$x' = vt$$

and then the range as a function of time is

$$r(t) = \sqrt{(vt)^2 + y^2 + z^2}$$

We usually simplify this by defining a range r_0 as the range at time 0, or the closest approach of the platform to the object:



Then

$$r(t) = \sqrt{r_0^2 + v^2 t^2}$$

What will be the measured phase of the signal as a function of time? Examination of the wave equation solution

$$\cos\left(\omega t - \frac{2\pi}{\lambda} x\right)$$

yield the relation of phase to ~~Ra~~ distance

$$\phi(t) = -\frac{4\pi}{\lambda} r(t)$$

where we have included an extra factor of 2 to account for two-way travel. In other words, the phase is simply the distance $r(t)$ expressed in wavelengths, doubled for round-trip travel, with 2π phase accumulating for each wavelength traveled. The minus sign is definitional and follows from the wave equation solution.

So what is the phase of the radar echo from our point as a function of time?

$$r(t) = \sqrt{r_0^2 + v^2t^2}$$

$$\boxed{\phi(t) = -\frac{4\pi}{\lambda}\sqrt{r_0^2 + v^2t^2}}$$

This equation is called the phase history of a scatterer at range r_0 and time $t=0$.

How does this relate to our previous discussion of Doppler frequency? Since

$$f = \frac{d\omega}{dt}$$

$$= \frac{1}{2\pi}\frac{d\phi}{dt}$$

then

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}\left[-\frac{4\pi}{\lambda}\sqrt{r_0^2 + v^2t^2}\right]$$

$$= \frac{-2}{\lambda}\frac{d}{dt}\sqrt{r_0^2 + v^2t^2}$$

Hence

$$f(t) = -\frac{2}{\lambda} \cdot \frac{1}{2} \frac{1}{\sqrt{r_0^2+v^2t^2}} \cdot 2v^2t$$

$$= -\frac{2v}{\lambda} \frac{vt}{\sqrt{r_0^2+v^2t^2}}$$

$$= -\frac{2v}{\lambda} \frac{x'}{r(t)}$$

$$= -\frac{2v}{\lambda} \sin\theta \sin\left[\phi(t)\right]$$

squint angle, not signal phase!

We also have a sign reversal because for positive time we are "looking" behind the radar.

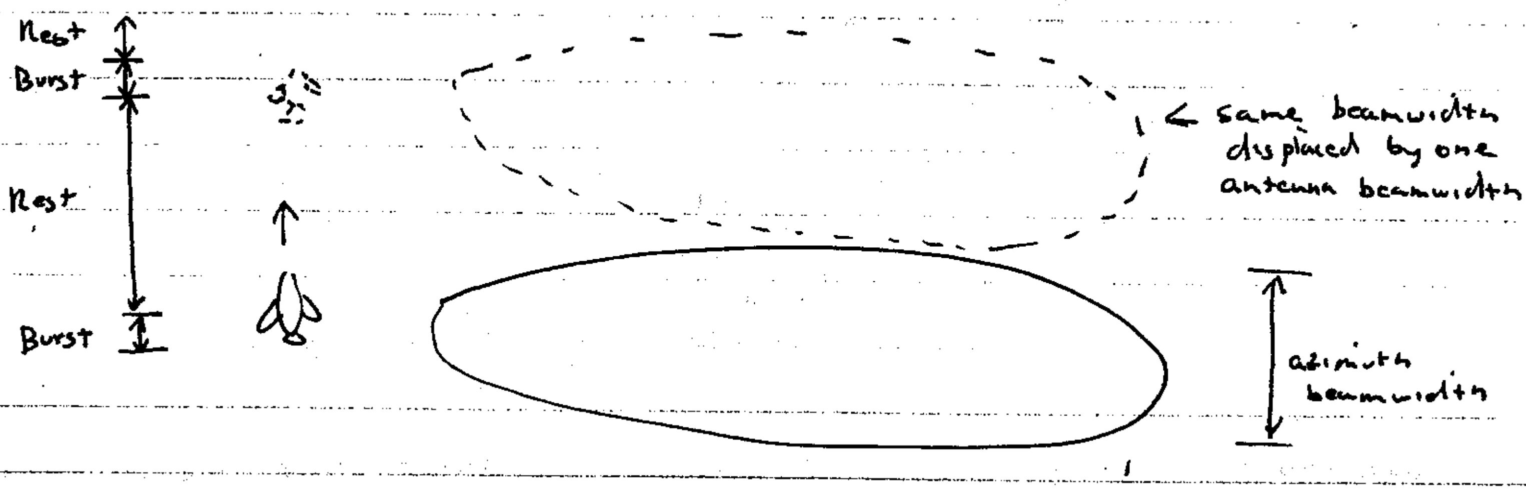
Therefore, if $\phi(t)$ (squint angle) does not change appreciably we can assume it is constant and use the Fourier transform method to generate along-track resolution. What is this value?

For now, let's use the following expression for the resolution of the unfocused algorithm.

$$\delta_{az,\,unfocused} = \sqrt{\lambda \cdot r}$$

(We'll be able to prove this later, when we discuss phase histories) Essentially we can send out pulses until we travel the distance according to the above relationship. We can now "design" an unfocussed processing algorithm for a radar system as an example to see how these various quantities interrelate.

Let's design an aircraft system operating at C-band, with an antenna length of 1 m. Here is our geometry from a viewpoint high above the radar:



We will send out a burst of pulse at the location shown in the solid line, process these to an unfocused image, and then wait until the platform moves to a new position displaced by an amount equal to our imaged area. We then repeat the entire process.

First, we need to know our imaged beamwidth. For a nominal ~~range~~ slant range r_o of 15 km:

$$\text{ground extent} = \frac{r_o \lambda}{l} = \frac{15000 \times 0.06}{1} = 900 \text{ m}$$

If the airplane flies at 200 m/s, our total cycle time is

$$\text{Cycle time} = \frac{900 \text{ m}}{200 \text{m/s}} = 4.5 \text{ s}$$

What is the range of Doppler frequencies we expect?

$$f_{max} = \frac{2v}{\lambda} \sin\theta \sin\phi_{max}$$

$$= \frac{2v}{\lambda} \sin(\epsilon q_{max})$$

$$= \frac{2v}{\lambda} \frac{x}{r}$$

$$f_{max} = \frac{2 \cdot 200}{0.06} \cdot \frac{450}{15000}$$

where we use 450 m and assume we want to see Dopplers in the range $\pm f_{max}$. Thus

$$f_{max} = 200 \text{ Hz}$$

Our total Doppler spectrum varies therefore from −200 Hz to +200 Hz, a bandwidth of 400 Hz. We consequently set our <u>pulse repetition rate</u>, or <u>pulse repetition frequency</u>, or <u>PRF</u>, to 400 Hz.

How about the number of pulses to use?

Our resolution is

$$\delta_{az} = \sqrt{\lambda R}$$

$$= \sqrt{0.06 \cdot 15000}$$

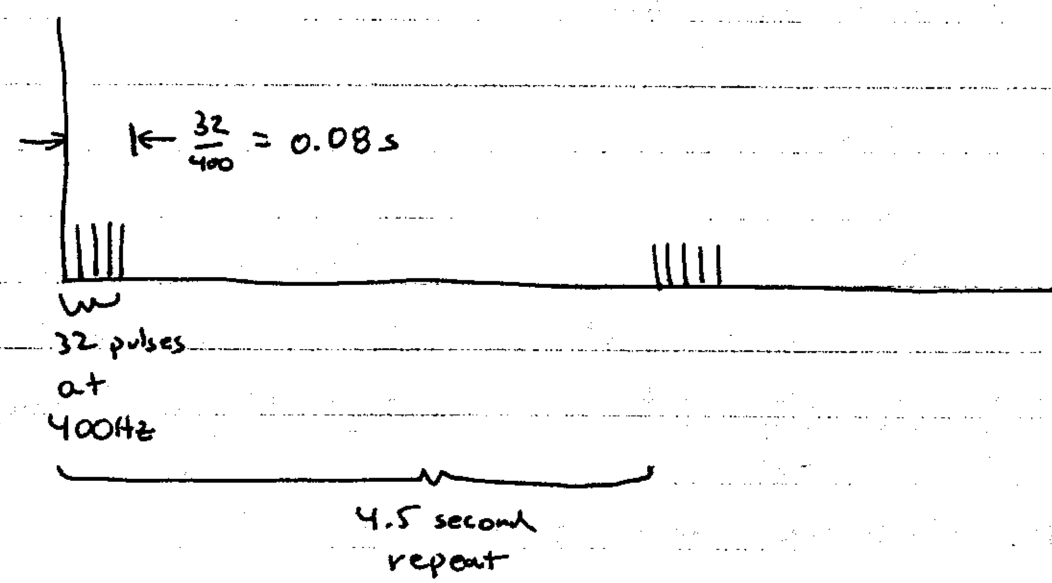
$$= 30 \text{ m}$$

Hence we expect to resolve

$$\frac{900}{30} = 30$$

resolution elements in azimuth. Increasing this to a next higher power of two to facilitate fft processing, we will transmit 32 pulses. Transforming these 32 values for each range bin yields the image for one particular point in time.

The time description of the radar looks like

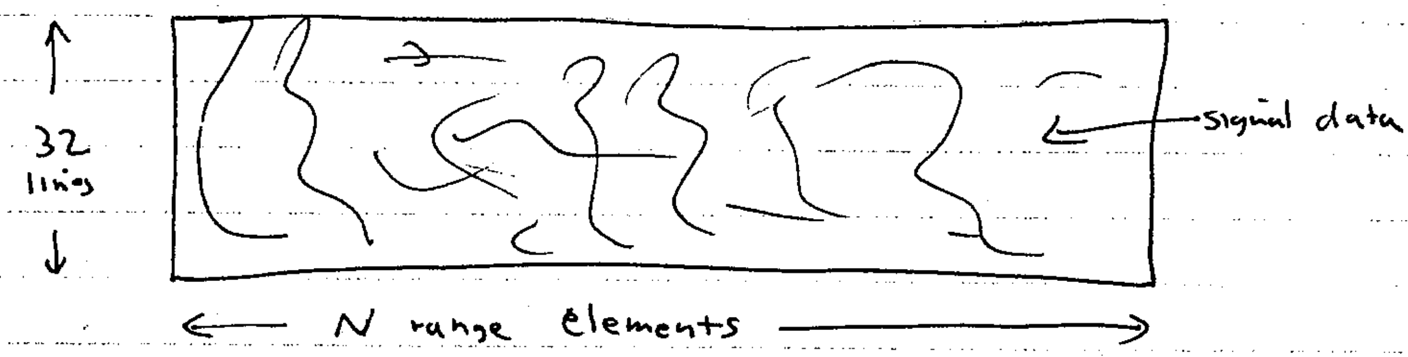


Let's see how much the platform has moved during our transmit interval:

$$\text{distance} = 0.08\,s \times 200\,m/s = 16\,m$$

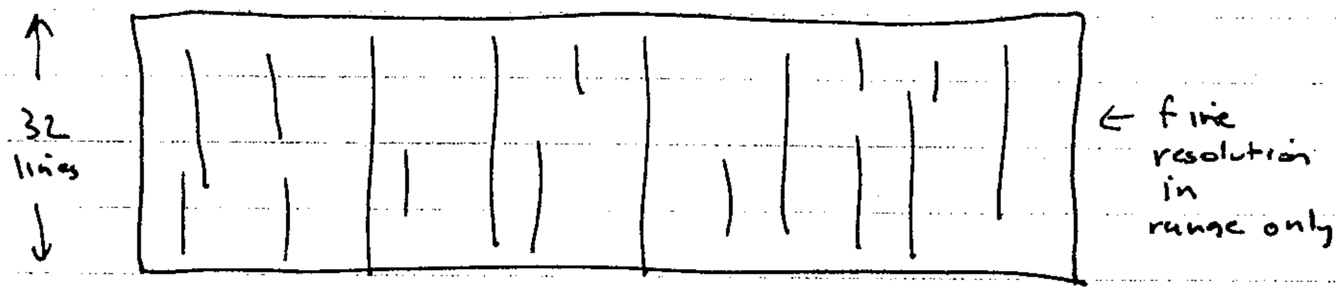
which is less than our resolution so we wouldn't expect any motion smearing.

Pictures of the processing steps:



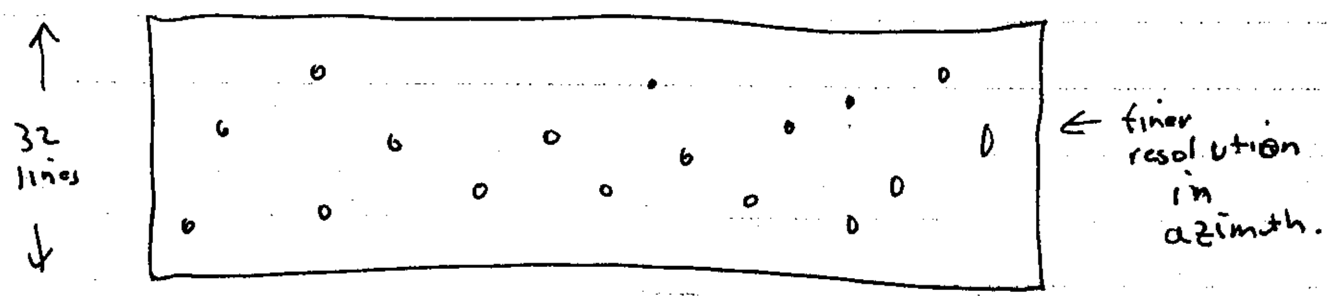
Original data

First we process in the range direction:



N range elements – chirp overhead

Then we transform each column in the data set:



N range elements – chirp overhead