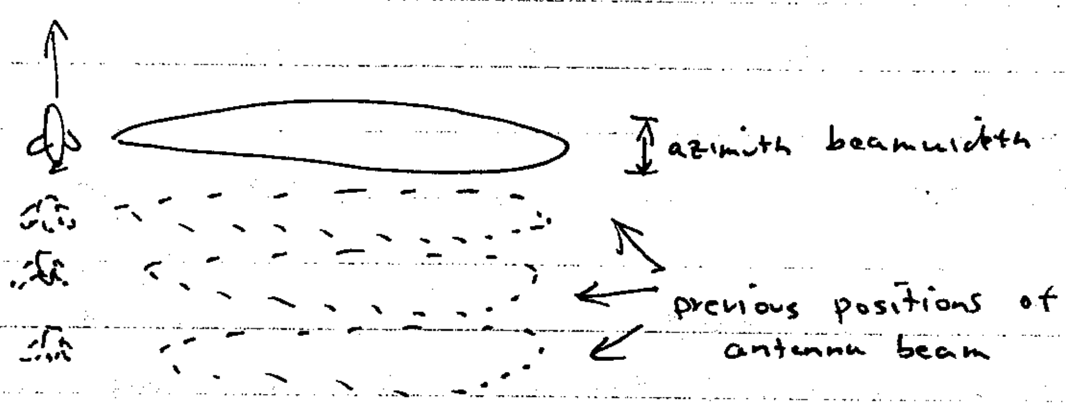
Azimuth resolution and image formation

Now we are ready to proceed to azimuth image formation, that is how we generate resolution in the other dimension. Let's begin by looking at a real-aperture system, as opposed to a synthetic aperture system.

Real Aperture Radar

In this case we have a platform flying along at constant velocity imaging a swath. Assume we have obtained fine range resolution via proper range modulation and processing.

Top view



If we were to pulse the radar occasionally, we could "stack" each range line, eventually forming an image of the surface. We need pulse the system only once each time the plane flies the width of the ground projection of the antenna beam, although we could pulse it much faster.

We know how to calculate SNR for this radar already. How does it perform in terms of resolution?

We cannot distinguish scatterers at different along track positions within the antenna beam.

Our azimuth resolution in a RAR is just the ground projection of the antenna:

$$\delta_{az\,(RAR)} = \frac{r\lambda}{l}$$

as before. Let's look at a few cases: (typical values shown)

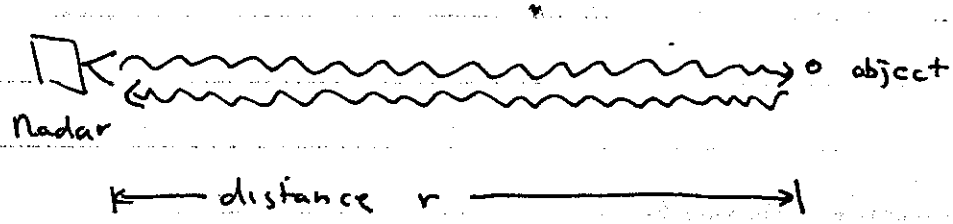
	λ	r	l	δ_{az}
Airborne L-band radar	0.24	15 km	1	3,600
Airborne X-band radar	0.03	15 km	1	450
Spaceborne L-band radar	0.24	800 km	10	19,200

While the X-band resolution is only $4\frac{1}{2}$ Stanford stadiums in extent, even that one is too large for most uses. Obviously the spaceborne systems are much too coarse for almost any practical situation. We need, therefore, to be more clever at exploiting the characteristics of the radar echo to generate finer resolution. (Recall that in range we have looked at meter-scale resolutions)

In order to do this, let's consider some of the properties of a radar echo.

Phase of a radar echo

Consider the relation between distance and phase in radar echoes:



Every time an EM wave propagates its wavelength its phase advances by 2π (a definition). Thus, we can relate range r to phase ϕ using the following

$$\phi = \frac{4\pi}{\lambda} r$$

If the range from an object to the radar is a variable function of time, $r(t)$, so is the phase

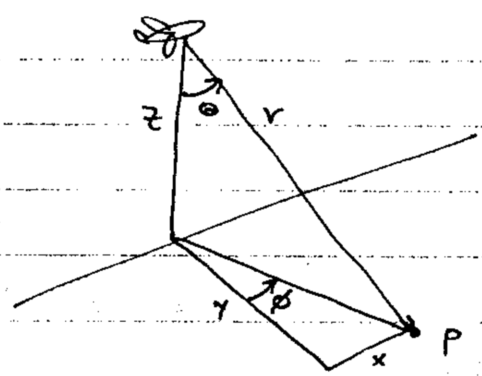
$$\phi(t) = \frac{4\pi}{\lambda} r(t)$$

Strictly speaking, if the radar signal is not monochromatic then the phase will be less well defined. But let's assume that the modulation (from the chirp) we put on the radar is small compared to its operating frequency, usually a good assumption. Then we can treat the signal as if all energy were concentrated at a single frequency.

For very wide-band or low-carrier-frequency radars, clearly this assumption is invalid. In that case what we do is to reformulate the problem in terms of ~~tide~~ time delays rather than phases. It is a parallel development to what we are doing here, and we won't repeat it as it is a specialized application.

Refresher on imaging geometry

Here is our generalized imaging geometry:

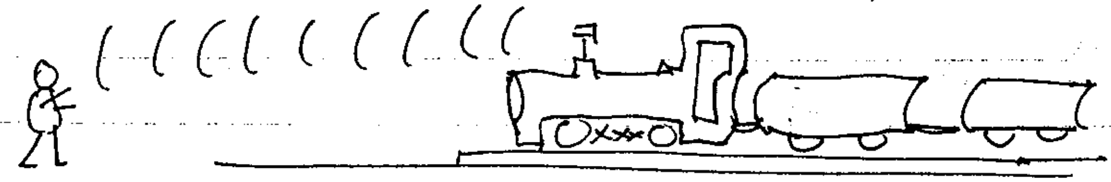


We'll need this in our investigation of the echo from various points in our illuminated beam.

We are going to separate our echoes along-track using the Doppler effect. Calculating the along-track Doppler component is the first step toward achieving higher azimuth resolution.

The Doppler effect

You have probably seen the Doppler effect and how it relates to sound echoes. This illustration probably looked something like this:



Because of the train's motion toward you, each wavefront is emitted a little closer to the previous waveform than we would expect from the wavelength. In fact the new wavelength λ' is shorter by the amount the train travels in the time required for one wavelength, or

$$\lambda' = \lambda - v \cdot \Delta t$$

$$= \lambda - \frac{v\lambda}{c}$$

where c here is the speed of sound rather than light. Then letting the speed of the train toward the listener be v:

$$\lambda' - \lambda = -\frac{\lambda v}{c}$$

or

$$\frac{\Delta\lambda}{\lambda} = -\frac{v}{c}$$

In the case of frequency $f = \frac{c}{\lambda}$, ~~we~~ we can restate the above as

$$\frac{c}{f'} = \frac{c}{f} - \frac{v}{f}$$

Hence

$$\frac{f'}{c} = \frac{f}{c-v}$$

$$\frac{f'}{f} = \frac{c}{c-v}$$

$$\frac{f'-f}{f} = \frac{c}{c-v} - 1$$

$$\frac{\Delta f}{f} = \frac{v}{c-v} \approx \frac{v}{c}$$ ← assume $v \ll c$, the non-relativistic Doppler relationship

$$\Delta f = \frac{v}{\lambda}$$

In the radar case, because we have two-way travel, a factor of 2 is involved. Also we can generalize the relation for an arbitrary velocity vector v for the target to the radar and a unit vector from the radar to the object u, and obtain

$$\boxed{\Delta f = \frac{2\, v \cdot u}{\lambda}}$$

where $v \cdot u$ represents the component of velocity toward the radar.