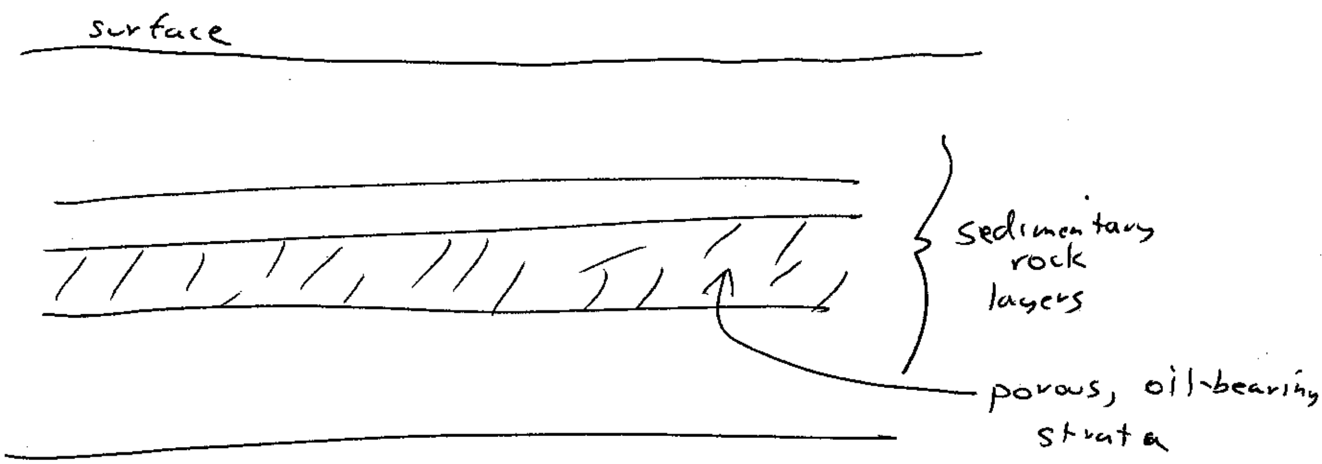
Imaging the Earth's Subsurface

Finally we are ready to begin to investigate some actual imaging systems. We'll start with an idealized system applied to measuring layers beneath the Earth's surface.

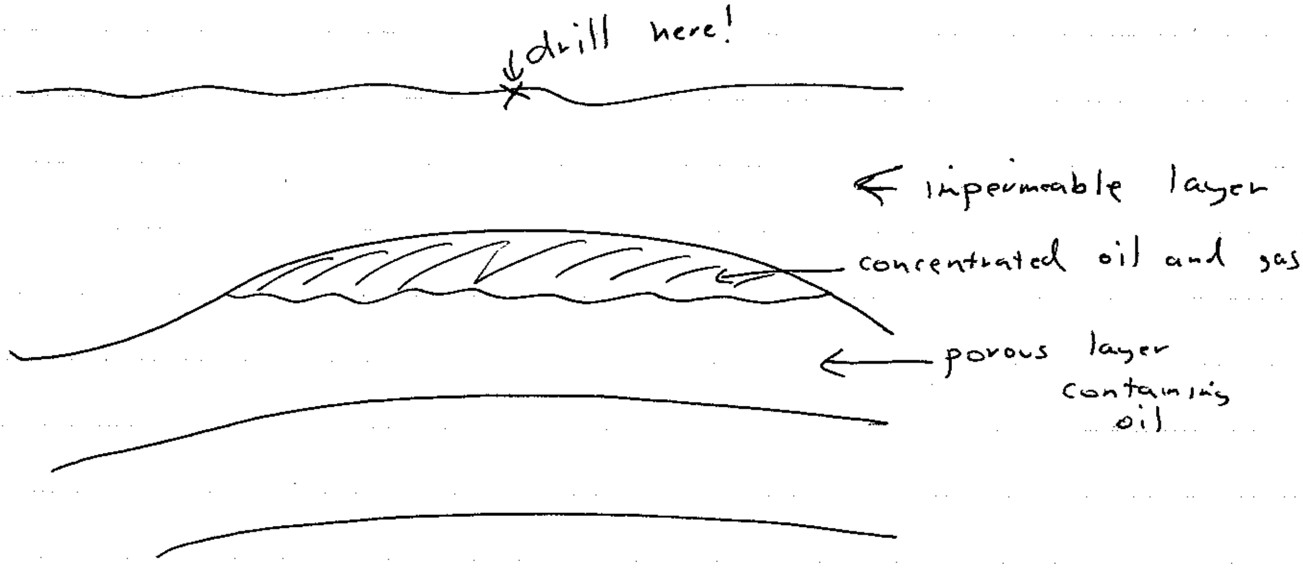
Why are we interested in this problem? Other than its intrinsic scientific interest, we can (and do) use such systems to find oil and gas below the surface.

Simple Earth structure



Over geologic time the Earth's crust becomes layered as successive deposits accumulate over time. These layers often include organic matter - dead animals and plants. In rocks of the proper age (~ Cenozoic and Mesozoic ages) the organic matter is converted to oil (and gas) with the suitable application of pressure and temperature. Older rocks probably had oil at one time but it has sinced been converted to another form or dissipated over time.

If the rocks remain plane-parallel to the surface, the ~~density~~ amount of oil per unit rock volume is often too low to remove efficiently, so the oil is too expensive to pump. But if the layers buckle under tectonic forces then local high and low spots appear, and as the oil migrates upward (usually) it collects in pockets of porous rock in stronger concentrations.

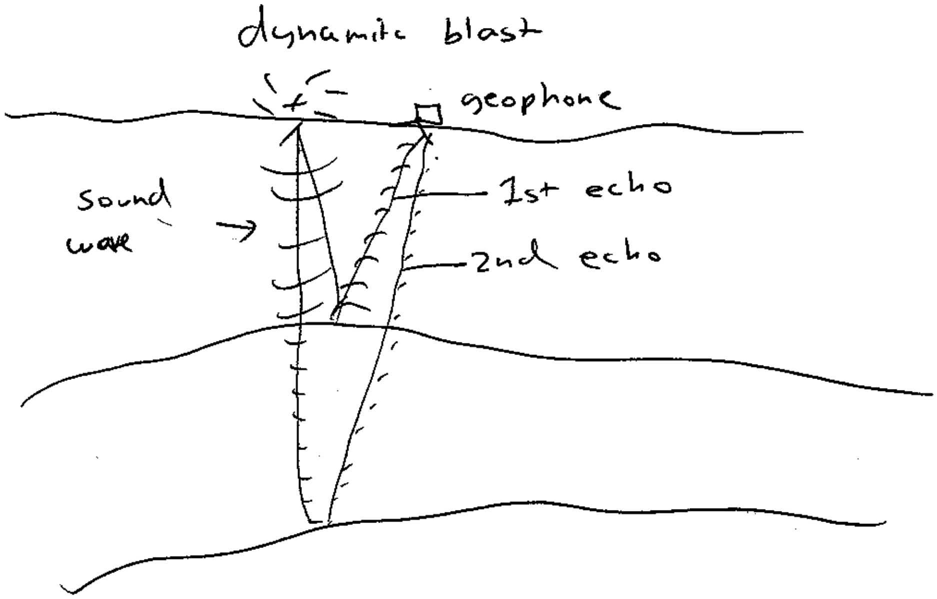


So if we could measure the layer depths, we could decide where to drill most economically.

How can we find the layer depths?

Seismic Shots

Suppose we have a source of sound and a detecting device. Typically we use dynamite for a source and a geophone (microphone) to listen. We perform the following experiment:



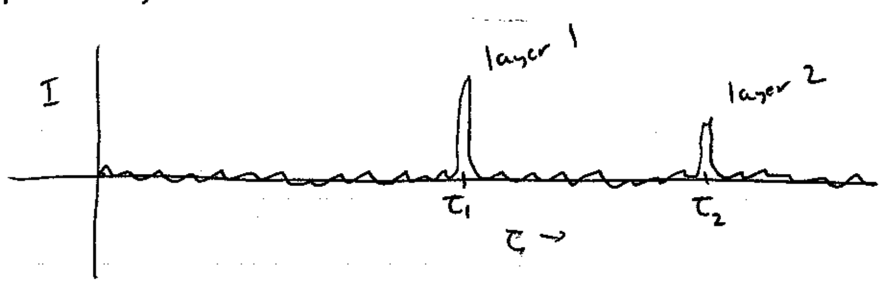
A large sound wave propagates from the dynamite, to the layer, and back to the geophone. If there are multiple layers each has an echo associated with it. The first echo comes from the top layer, then each successively deeper layer.

How do we get the depth? If we know the velocity of sound in the rocks, the depth d is simply

$$d = \frac{V \cdot \tau}{2}$$

where τ is the time delay of the echo and the factor of 2 comes from 2-way travel (down and back). If there were 2 layers, the intensity of sound at the geophone would have 2 peaks.

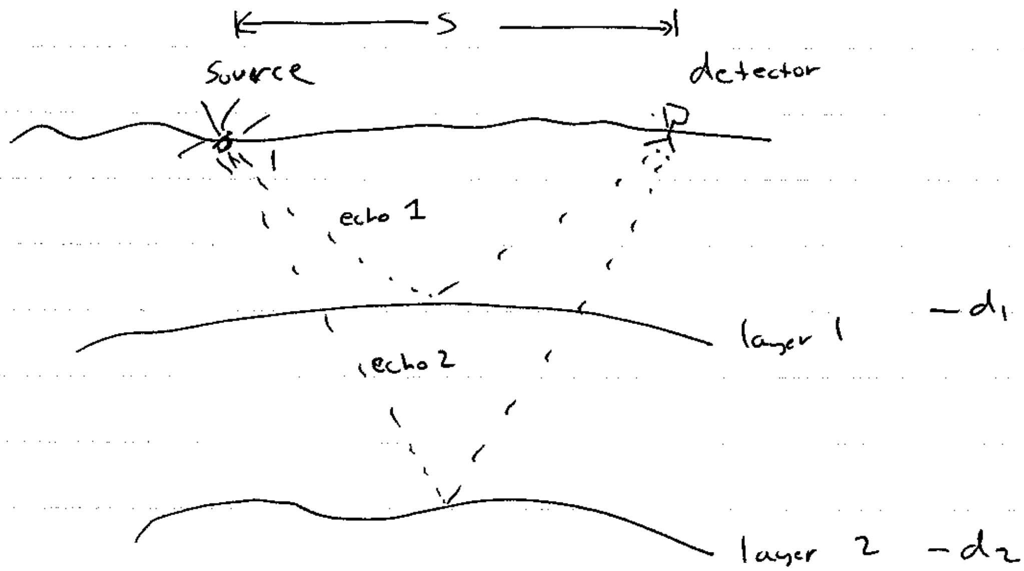
Geophone signal vs. time:



The depths of the layers d_1 and d_2 would be

$$d_1 = \frac{v\tau_1}{2} \qquad d_2 = \frac{v\tau_2}{2}$$

The above holds if the source and geophone are located close together on the surface. Suppose now that the source and detector are separated by a spacing s. How does that change the geometry?



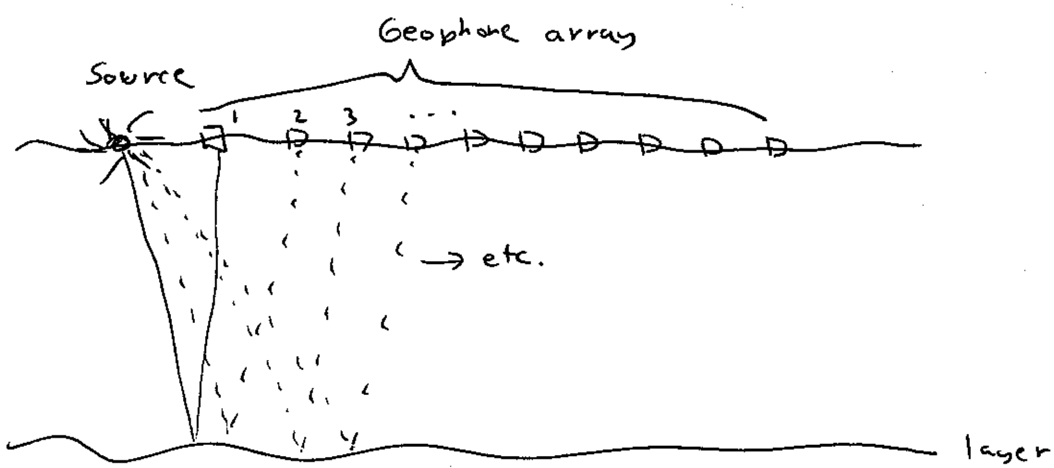
The total time for the echoes now is related to depth by

$$d_1 = \sqrt{\frac{v^2\tau_1^2}{4} - \frac{s^2}{4}} \qquad d_2 = \sqrt{\frac{v^2\tau_2^2}{4} - \frac{s^2}{4}}$$

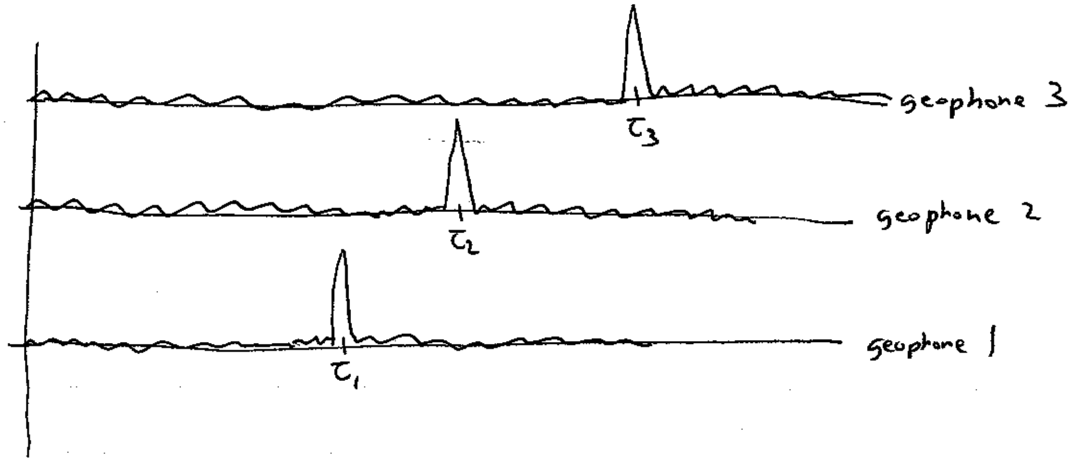
Why would we want to have source and detector separated? Since each echo can be quite faint, we'd like to be able to make several measurements and average them. There are several reasons we wouldn't just repeat the experiment at zero separation many times:

- Multiple dynamite blasts are dangerous
- Multiple blasts are expensive
- Multiple blasts destroy the source region leading to poor coupling into the ground
- There are repetitive coherent noises that won't average away

So what is often done is ~~that~~ a single shot exciting a series of ~~geo~~ geophones is used, as in:



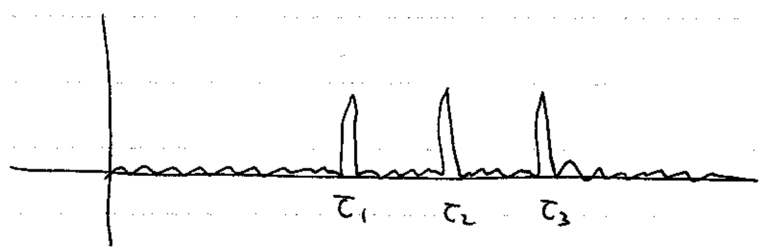
If the layer is locally flat, the time delay of the echo depends only on the separation of the source and detector, as well as the depth. Traces might look like



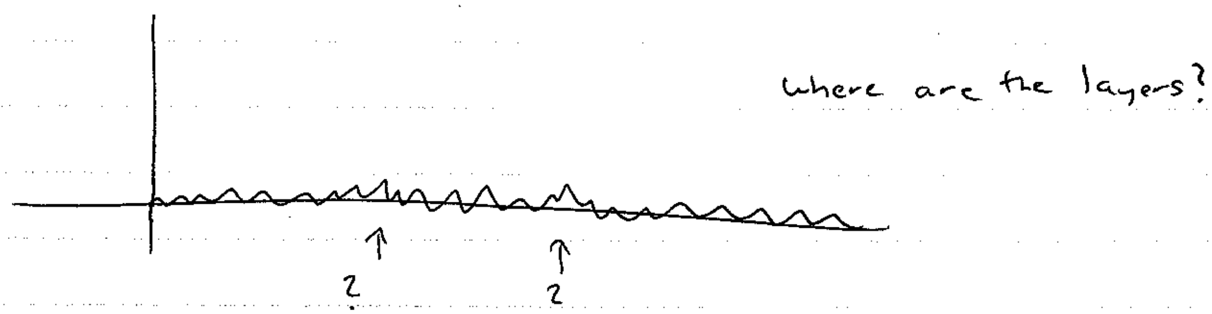
where the depth d is related to each geophone spaced s_i and results in an echo at τ_i:

$$d = \sqrt{\frac{v^2 \tau_i^2}{4} - \frac{s_i^2}{4}}$$

Thus multiple estimations of d are available. But we wanted to combine measurements to improve SNR. Consider summing the 3 traces we had above:



This would be ok if the SNR were high enough to identify each peak. But if SNR were low initially the sum would look more like

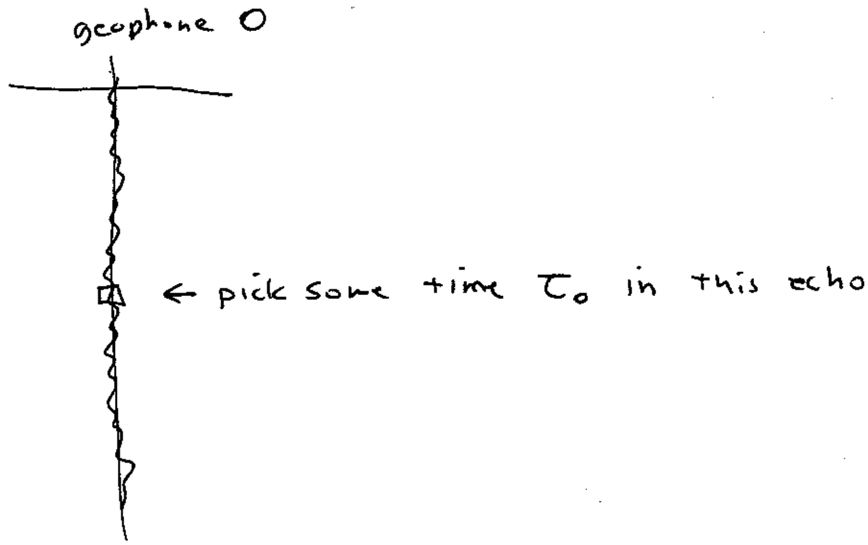


How to resolve this? We know explicitly from the geometry how the τ_i's are related if we know the depth and also s_i. Thus we can modify each trace in time assuming we have an echo at each possible time so that the echoes lie on top of each other. Then if we sum the echoes will tend to enhance each other.

This method of summing is called <u>stacking</u> in the oil industry.

How do we implement such an algorithm? First we need to define a reference to compare all the other echoes with. It is convenient to use the geophone at zero offset as a reference.

Let's also plot our time series as down rather than across to suggest depth in the Earth.



What depth would this time correspond to if we have $s_0 = 0$?

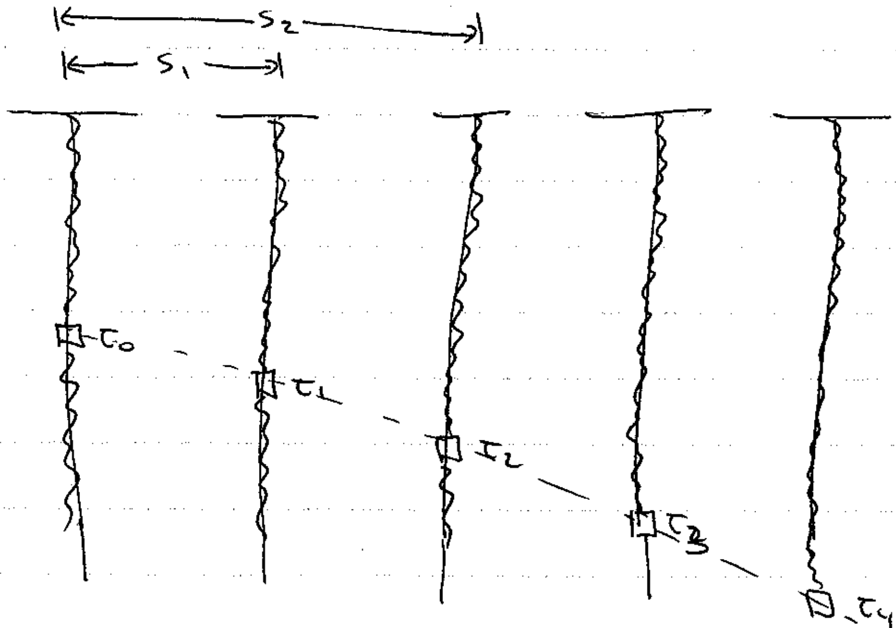
$$d = \frac{v\tau_0}{2}$$

If the layer depth is constant at d, where would an echo from that layer appear in a displaced geophone at s_i?

$$d = \sqrt{\frac{v^2\tau_i^2}{4} - \frac{s_i^2}{4}} \quad \text{or} \quad d^2 + \frac{s_i^2}{4} = \frac{v^2\tau_i^2}{4}$$

$$\Rightarrow \sqrt{\frac{4d^2 + s_i^2}{v^2}} = \tau_i$$

Hence for a layer of depth d:



The points τ_i lie on a hyperbola defined by

$$\tau_i = \sqrt{\frac{4d^2 + s_i^2}{v^2}}$$

So, if we compensate for this hyperbolic "trajectory" we can add the traces to improve SNR. This is the <u>stacking</u>.

More formally, let each echo time series at a geophone g_i be $g_i(t)$. We form the sum

$$\text{sum}(t) = \sum_{i=1}^{N} g_i\left(\sqrt{\frac{4d^2 + s_i^2}{v^2}}\right)$$

for each layer at depth d. If we reference our result to the ~~depth~~ time in the zero-offset geophone, then

$$d = \frac{vt}{2} \quad \text{or} \quad \frac{2d}{v} = t$$

so that

$$\text{sum}(t) = \sum_{i=1}^{N} g_i\left(\sqrt{\frac{v^2t^2 + s_i^2}{v^2}}\right)$$

or

$$\text{sum}(t) = \sum_{i=1}^{N} g_i\left(\sqrt{t^2+\left(\frac{s_i}{V}\right)^2}\right)$$

Repeating this operation for all t gives our estimate of depths.

Practical details

Now that we have a method for stacking, how do we implement it in practice?

First, we don't usually start recording our echoes at the time of the dynamite blast. We're generally interested in layers fairly deep under the surface and it would be wasteful of memory to record data from the topmost regions. Also the blast will saturate the sensors for some time and the near-surface data will be corrupted anyway.

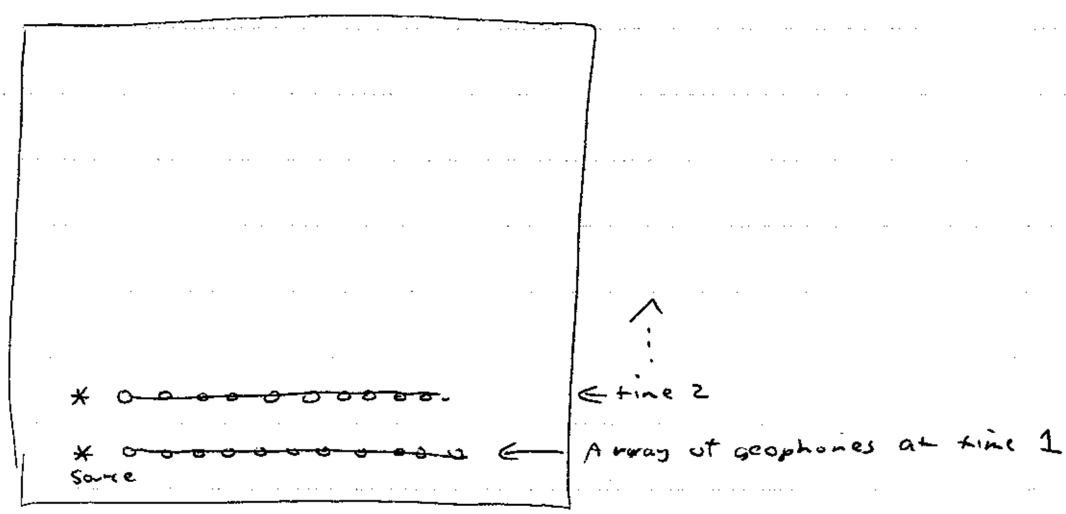
Also, our data will be from a sampled system of necessity. The sample rate will be set by the thickness of each layer and its precision in location, and will be as low as we can set it to save memory.

Another problem is that the velocity typically changes with depth. Deeper rocks under more pressure have higher velocities ~~st~~ than shallower rocks. Hence we need an "Earth model" to properly infer depths and structure.

Another issue is that while the measured echo is sampled at known points for the reference, the values at which we want to know the echo (according to the hyperbolic trajectory) are not usually coincident with the times in the arbitrary echoes. Thus some interpolation is required.

Measuring a profile (or section)

Typically we want to create an "image" of the layers under the surface rather than just a single measurement stack. There are several geometries for this, among them might be



where the array of receivers and the source are moved up the area to be surveyed. Here a linear section would be traced out upward under the assumption that the depth of each layer is constant under the array from left to right.

Another geometry would be to rotate the array 90° and move it in the same direction it is aligned with. This has the advantage of not requiring a constant layer depth below the array, but additional processing is needed in that case. Here the array is usually moved by an amount equal to the geophone spacing between shots, so that each geophone position is occupied many times. This "redundant" data set allows reconstruction of varying profiles if what is called "migration" is added to the code. Migration accounts for movements in depth of layers but we won't detail it here.

Note that for a sonar system in which a ship tows an array of hydrophones this latter geometry is a natural choice: