Coherent and Incoherent Images

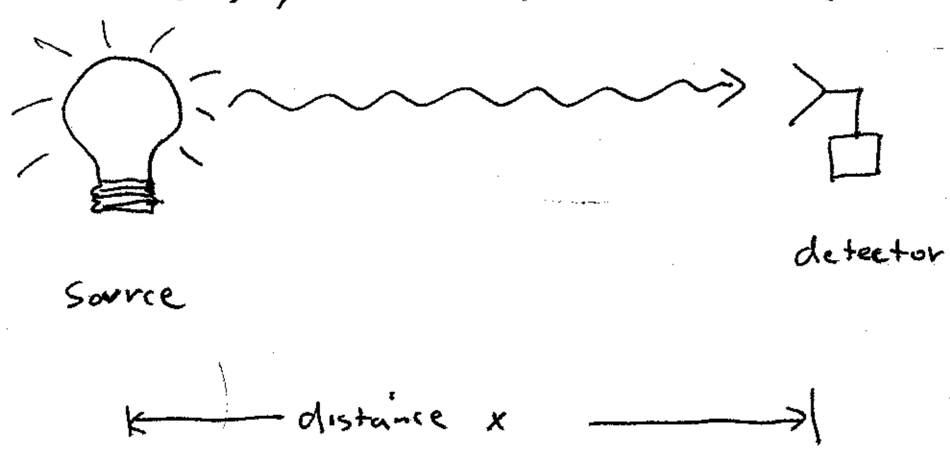
Whether or not an imaging system is coherent has significant implication for the images produced, and how they are to be interpreted.

What do we mean by coherent? For our purposes, we will state that if we can measure, even in principle, a meaningful phase estimate of our illuminating, reflected, or emitted radiation then the system is coherent. If only the amplitude or power of the radiation can be reliably determined we will denote that system as incoherent.

Typically an incoherent beam of radiation possesses wide spectral bandwidth and/or a diverse set of polarization states. We can think of an incoherent radiation stream as composed of many individual components, each coherent, added together. The resulting radiation would then have a phase that is essentially random, being the sum of many independent waves.

Coherent and incoherent sources

Consider a single, narrowband source and a detector:



Coherent / Incoherent Image Properties

	Incoherent	Coherent
Image represented by:	Intensity at each point (I)	Complex field quantity $c = (rv_1, rv_2)$ where $rv_i = N(0, \sigma^2)$ and $\sigma^2 = \langle I \rangle$
	Amplitude at each point $(\sqrt{I})$	
Quantity displayed	I - intensity image	$\lvert c \rvert^2$ - intensity image
	$\sqrt{I}$ - amplitude image	$\lvert c \rvert$ - amplitude image
		$\tan^{-1}\left(\frac{\text{imag}(c)}{\text{real}(c)}\right)$ - phase image

The information in a coherent image may be in the phase, the amplitude, or both. In an incoherent image only the intensity has any physical meaning.

Suppose the source emits radiation at a constant frequency ω.

source amplitude $\propto e^{j\omega t}$

This is a complex quantity of unit intensity and phase ωt. If this represents the amplitude of a wave, after propagation to the detector it can be represented as proportional to

$$\frac{e^{-j(kx-\omega t)}}{x}$$

where kx is an additional phase due to travel along x. The quantity k is the wavenumber $\frac{2\pi}{\lambda}$, λ wavelength. Note that x appears also in the denominator showing that the amplitude also decreases as the wave propagates.

We usually omit explicit time dependence of the wave since can always put it back in at the end, and it is a constant multiplicative factor in what follows. In addition for simplicity we'll assume that all waves travel the same distance x and are attenuated, or reduced in intensity similarly.

Thus, we write the complex amplitude of the wave at the ~~decte~~ detector as

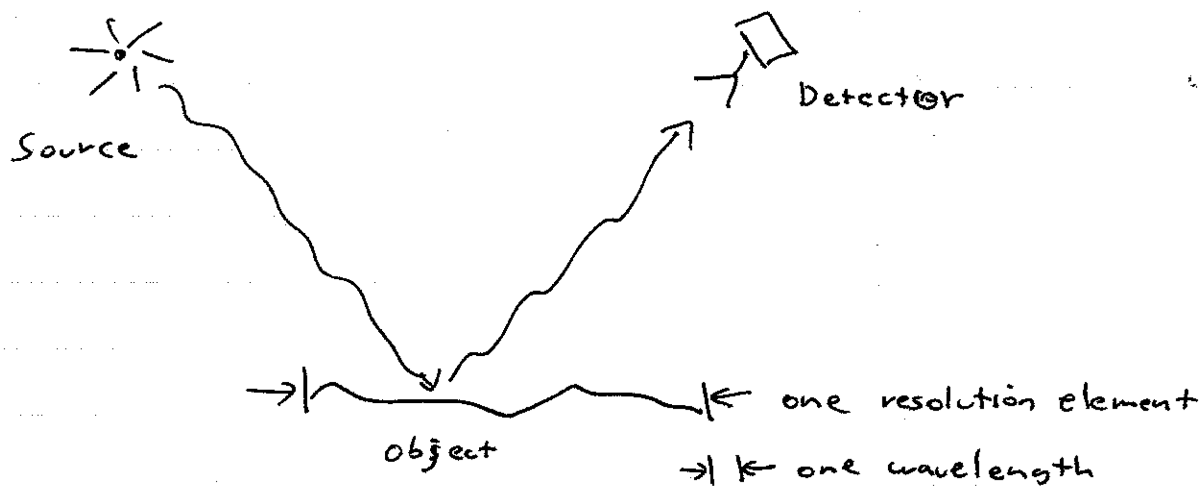
$$e^{-jkx}$$

This wave has unit amplitude and phase kx. Both quanities are well defined, the phase varies linearly with x, and thus we would call this single wave coherent.

In contrast, consider a source emitting many waves at different wavelengths. Then the complex amplitude of the wave at the detector would be the sum over all waves k_i

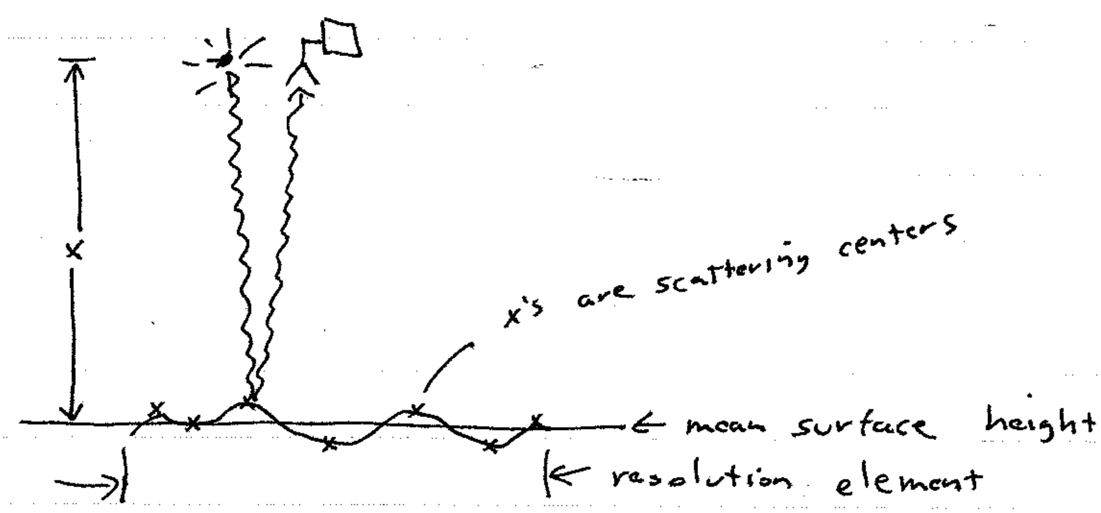
$$\text{sum} = \sum_{i=1}^{N} e^{-jk_i x}$$

What does this mean for imaging? Let's look at a typical imaging geometry:



Again, we have many scatterers per resolution element.

For simplicity, let's alter the geometry slightly and then evaluate the amplitude of the wave at the detector.



Note that because of the intrinsic roughness of the surface, the several scatterers within a resolution element are at slightly different distances from the source and detector. More formally, if each scatterer is at distance x_n from the source/detector pair, for a given wave at frequency f the signal at the detector from the nth scatterer will be

$$s_{nth\ scatterer} = e^{-j 2\pi \frac{f}{c} x_n}$$

where we have used the usual definitions

$$f = \frac{c}{\lambda}, \quad k = \frac{2\pi}{\lambda}$$

c = speed of light,
λ = wavelength
k = wave number
f = frequency

Summing over **N** scatterers,

$$s = \sum_{n=1}^{N} e^{-j \frac{2\pi f}{c} x_n}$$

Now, if $|x_n - x| \ll \lambda$, all the waves will have essentially the same phase and will add to form a large amplitude. But, if the variations in length $x_n - \underline{X}$ are comparable to or larger than λ, the waves will have random phases. Then the sum can be large, if the waves tend to add in phase, or small, if they tend to cancel each other out.

~~Pr~~

This is the coherent imaging case.

Let's evaluate the statistics of the wave at the detector, specifically the statistics of the power (intensity) of the wave assuming the surface is "rough" in wavelengths $[\,|x_n - x| > \lambda, \text{all } n\,]$. If N is large, say > 10, we will find that the power in the wave, or its intensity, i, defined by

$$i = s \cdot s^*$$

is exponentially distributed, with its mean equal to its standard deviation.

How about the incoherent case?

Now we will have to measure the sum of intensities at many different frequencies, all from the same set of scatterers. Because the frequencies are different, we can simply add the intensities rather than the field (complex) quantities. Using the above notation, letting i_m be the mth frequency power measurement, the total intensity i_{total} is

$$i_{total} = \sum_{m=1}^{M} i_m = \sum_{m=1}^{M} s_m s_m^*$$

where $$s_m = \sum_{n=1}^{N} e^{-j\frac{2\pi f}{c} x_n}$$

If we were to evaluate the statistics now of i_{total}, we'd find that the standard deviation will be less than the mean, and will get smaller ~~as~~ (w.r.t. the mean) as

M increases.

In fact, we can define a measure of detectability as the ratio of standard deviation to mean:

$$\text{detectability} = \frac{\sigma}{\mu}$$

If $\sigma = \mu$, as in the coherent case, detectability by this measure is 1, or 100%. In other words, the error on the measurement is as large as the measurement itself. For a large number of frequencies M, as in broadband illumination, detectability decreases significantly and in fact is approximately

$$\text{detectability} \approx \frac{\sigma}{\sqrt{M} \cdot \mu} = \frac{1}{\sqrt{M}} \quad \text{if } \sigma \simeq \mu \text{ for a single observation}$$

So if we had 100 different λ's then we improve the accuracy of our measurement by a factor of 10.

<u>How many λ's possible</u>?

Clearly the number of frequencies depends on the bandwidth of the radiation. It also depends on how accurately we can measure frequency. A rough guide is that we can determined f to an accuracy of $\frac{1}{\tau}$, where τ is the integration time in the detector. Hence for a signal of bandwidth B, observed for a time τ,

$$M \approx \frac{B}{1/\tau} = B\tau$$

or $M \approx$ the time-bandwidth product of the signal.

Example: White light from the sun in a camera

Bandwidth: visible spectrum $\approx 0.3\,\mu - 0.7\,\mu$ wavelength
or frequency from $4 \times 10^{14} - 10^{15}$
$= 6 \times 10^{14}$ Hz

Time: Aperture time typically $\frac{1}{500\text{th}}$ second $(\frac{1}{2} \times 10^{-3}\ s)$

$$M = 6 \times 10^{14} \times \frac{1}{2} \times 10^{-3} = 3 \times 10^{11}$$

Hence the detectability is about $\frac{1}{\sqrt{M}} = 1.8 \times 10^{-6}$. In contrast to the coherent case of 100% variation, this is about 2×10^{-4} %. Thus ordinary optical images don't exhibit much "speckle".

Examples of coherent and incoherent systems

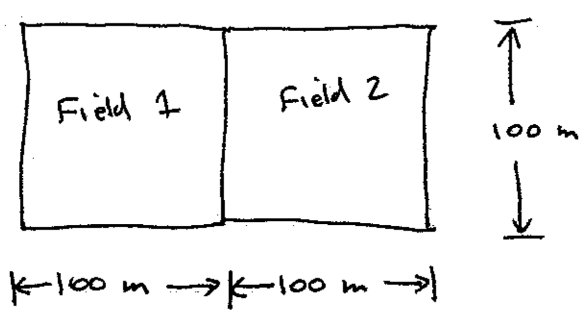
Incoherent	Coherent
Camera - film under natural light	Laser system
Vidicon under natural light	Acoustic Doppler imager
X-ray, broadband	Radar
Some medical tomography	

How about others? Why are the above in the columns they are in?

Looks

When we analyze/process coherent images, it can be very hard to identify objects because of speckle modulation of the signal. But we can improve detectability by artificially creating a "broadband" signal by adding multiple observations of a single scene, usually by averaging adjacent resels incoherently. These multiple observations are equivalent to looking at the scene with multiple frequencies.

Suppose our scene consists of two fields, and we wish to determine the relative brightness of each using a coherent imaging system.



Start with a 100 m resolution system. How many resels will we have for each field?

With one resel, the error in each measurement will be 100 % of the observed brightness. A comparison of the results from each field is essentially meaningless.

Now, repeat the experiment with a 10m resolution system. Here we will obtain 100 measurements ($M=100$)

for each field. The detectability measure now is about

$$\text{detectability} \approx \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{100}} = \frac{1}{10}$$

so the brightness for each field is about 10% error instead of 100%. Thus if the fields differ by about 10% in brightness we can be more sure of our ability to distinguish them.

<u>Tradeoff between resolution and looks</u>

Thus we see that there is a tradeoff between system resolution and our ability to distinguish brightnesses. We can view this in one of two equivalent ways:

1) For a given system, decreasing the output resolution by averaging adjacent pixels increases our ability to measure brightness differences

2) To distinguish fields of a certain size, increase initial resolution until each field is covered by enough resels to permit the brightness accuracy that is needed.

Simulation of coherent images

For class purposes we will at times need to generate simulated coherent images. We have already discussed their statistics, and now we consider how we might create a simulated image on the computer.

We know that a single pixel in a complex single-look image results from the amplitude or power of a 2-dimension Gaussian random variable c:

$$c = (n_r, n_i)$$

where n_r (real part), n_i (imaginary part) are ~~complex~~ Gaussian random variables of typically zero mean, and with variance equal to half the average intensity of the pixel. Then the average intensity of c is twice the variance of each n_r, n_i.

The intensity of c, $|c|^2$, is exponentially distributed with both mean and standard dev. equal to the average power of the pixel.

The amplitude of c, $|c|$, is Rayleigh distributed with its mean equal to $\sqrt{\frac{\pi}{2}}\,\sigma$, and variance $\frac{4-\pi}{2}\cdot\sigma^2$, where σ is the standard deviation of each of n_r, n_i.

→ So, if we know the intensity of each point in the image we can use that information to govern the statistics of the simulated image. ←

Steps in image simulation

1. The basic step is to realize that the image is fundamentally

derived from combinations of Gaussian random variables. Thus we need first to generate these easily.

Most computers have built-in functions to create random numbers uniformly distibuted between 0 and 1. How do we go from here to form normal rv's?

We could define an appropriate transformation we could apply to the uniform rv, or use Monte Carlo techinques to eliminate certain draws from the uniform generator. Much easier, though, is to use the central limit theorem, which states that the sum of many draws from almost <u>any</u> distribution tends to the normal distribution, with a mean equal to the sum of the means and variance equal to the sum of the individual variances.

What is the mean and variance of the uniform [0,1] distribution?

$$E(rv) = 0.5$$
$$Var(rv) = \frac{1}{12}$$

Hence, adding 12 uniform draws results in a Gaussian with unit variance and $\mu = 6$. Subtracting 6 gives $N(0,1)$.

Our formula is then

$$rv_{gaussian} = \sum_{i=1}^{12} rv_{uniform} - 6$$

2. The second step is to create 2-D gaussians

$$\text{complex gaussian} = (rv_{gaussian}, rv_{gaussian})$$

Each is one of our Gaussian draws

3. The total power in the complex gaussian is now 2, since the variance in each of the real and imaginary channels is unity. What remains is to use the intensity of each pixel ~~as~~ either the variance or standard deviation of the complex gaussian, as appropriate, and draw one complex gaussian rv to represent that pixel. Either the magnitude or magnitude squared, again as appropriate, may be displayed. The resulting image will now exhibit the "speckle" of a coherent image.

4. Finally, if multiple looks are desired, for each pixel several draws to the gaussian complex rv generator can be made, and the powers for each added to generate a multiple-look image.