

Homework # 5

Due next Wednesday

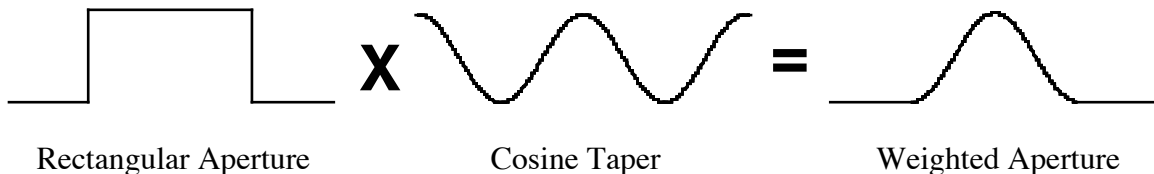
1. Consider the function $f(x,y) = \exp[-\pi(x^2/a^2 + y^2)]$, an elongated Gaussian hump. This function represents an imaging device (an example might possibly be your eye) with astigmatism. Find a function that can be convolved with $f(x,y)$ to result in a circularly symmetric response, thus correcting the astigmatic behavior of the original system.

2. Consider an antenna with a rectangular aperture function $\text{rect}(x)$. The aperture is thus 1 meter across. Assume a wavelength of 10 cm.

a. Calculate and plot the power pattern of the antenna using the Fraunhofer approximation. Label the x-axis in degrees and the intensity in dB.

b. What is the “peak sidelobe ratio,” the ratio of the highest part of the pattern to the greatest sidelobe, in dB ?

c. Show that tapering the aperture by multiplication with a cosine improves the sidelobe ratio. What is the peak sidelobe ratio for the tapered aperture ? Assume the tapering function goes to zero at the edges of the aperture, as shown in the following picture:



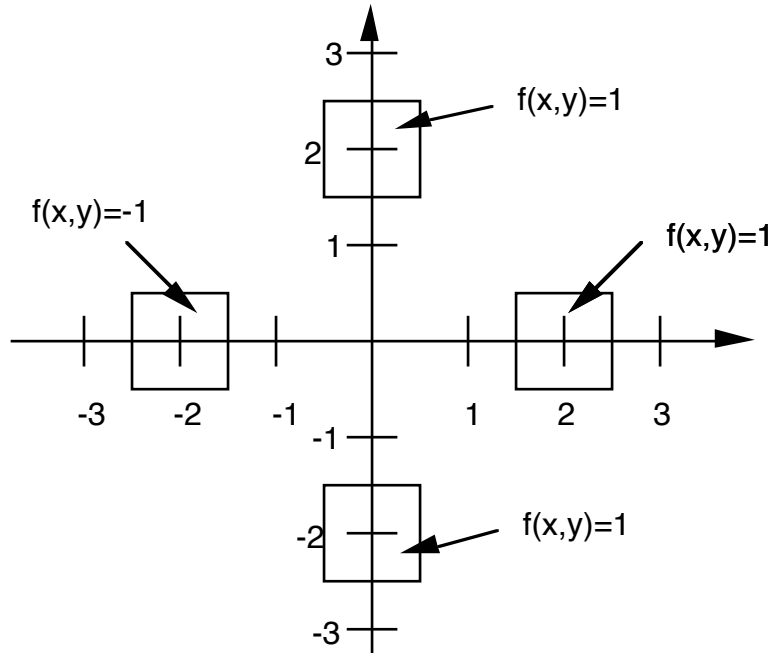
d. Using the convolution theorem, show that multiplication by the cosine results in sidelobe cancellation. Do this by illustrating what parts of the unweighted pattern cancel other sidelobes in the pattern.

3. Assume that you have a circular antenna with an aperture 1 m across. You use this antenna to transmit microwave energy with a wavelength of 25 cm.

a. Plot the intensity (power) pattern of the antenna, labeling the axes in degrees and the intensity in dB.

b. Weight the antenna using a cosine function dependent on radius, with the cosine function tapering to zero at the edges of the aperture. Now plot the intensity pattern as in (a). How do these sidelobe levels compare with the unweighted aperture? How much suppression, in dB, of the first sidelobe do you see?

4. A function $f(x,y)$ is defined as shown below. The function is zero except over four unit squares, and has magnitude +1 over three of the squares and -1 over the fourth as indicated.



a. Calculate the Fourier transform of $f(x,y)$ analytically and numerically using fft methods. Create a discretely sampled version of the analytical solution that matches the numerical result point by point. Display each as a tiff image.

b. Calculate the difference image between the two methods. Display as a tiff file that gives the difference directly and also as a “percentage error” image.