EE262 **Handout # 13**

Homework # 3

Due Thursday, Jan. 28, in class or in box by 4:00 PM

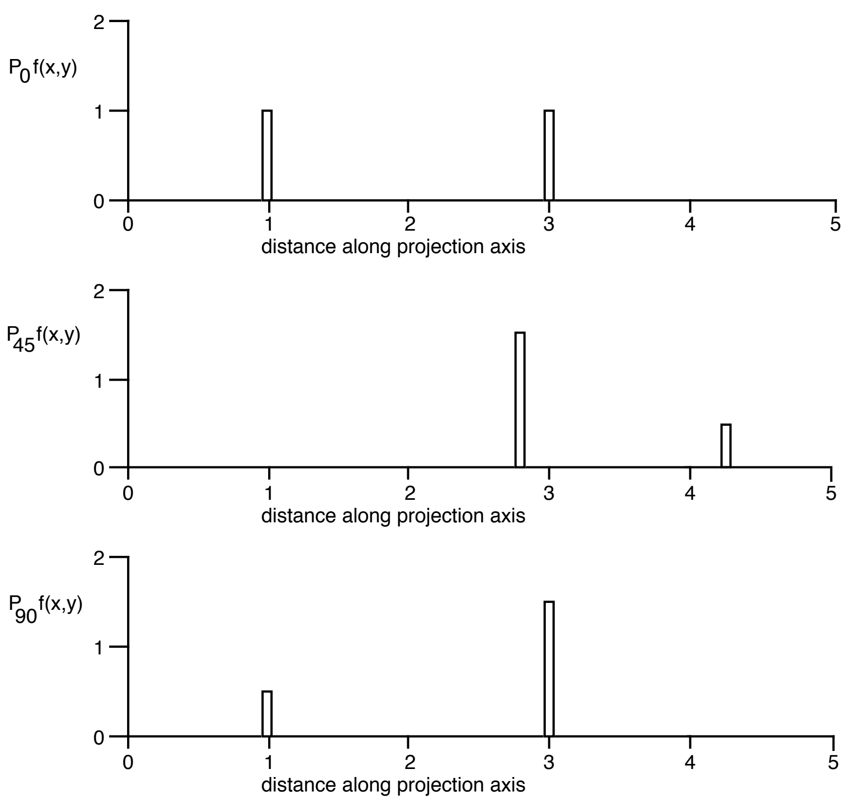
1. Consider the function $f(x,y) = {}^2\text{rect}(x-2,y-3) + 3 \bullet {}^2\text{rect}(x-2,y-1)$.

a. What is the projection of f(x,y) along a line oriented 0˚ (parallel) to the x-axis ? (In our notation in class this would be P_0 f(x,y).)

b. What is the projection of f(x,y) along a line oriented 90˚ to the x-axis ? (P_{90} f(x,y))

c. What is the projection of f(x,y) along a line oriented 45˚ to the x-axis ? (P_{45} f(x,y))

2. Suppose we have three scans (projections) of an object acquired at 0˚, 45˚, and 90˚, respectively, as follows:



Using back projection, reconstruct an estimate of the original function f(x,y).

3. Show that the strength of the impulse of a function f(x) is inversely proportional to its gradient evaluated at a root of the function, that is,

$$\delta(f(x)) = \frac{1}{|f'(x_0)|}\,\delta(x-x_0)$$

where x_0 is a root of f(x).

4. Evaluate the line integral along a circle of radius R of a surface f(x,y) = cos (πx) for the following cases:

a. The circle is defined by the ring impulse $\delta(r-R)$.

b. The circle is defined by the ring impulse $\delta(r^2-R^2)$.

c. The circle is defined by the ring impulse $\delta(\tan^{-1}(r-R))$.

d. How about if the circle is defined by the ring impulse $\delta((r-R)^2)$? Can you explain this result using properties of the function within the $\delta(\ )$?

5. Download the image file "hw3p5image" from the class web homework area. This is a byte file with a picture of dimension 480 by 640. Assume that the origin of the coordinate system is at pixel 241, line 321, referenced to the upper left hand corner of the image.

Sample the image using the following impulse functions, that is, evaluate the integral of the product image and the impulse function over all space. The units of the arguments for the impulse functions are in pixels.

a. The line impulse ${}^2\delta(x)$.

b. The line impulse ${}^2\delta(2y)$.

c. The line impulse ${}^2\delta(x-y)$.

d. The circular impulse ${}^2\delta(r-100)$.

6. Consider a three-dimensional impulse $\delta(R - (x^2+y^2+z^2)^{1/2})$.

a. What is the strength per unit *area* of this impulse? What is its total strength integrated over all space?

b. Suppose a light source is located at the origin of the coordinates and that it emits light isotropically at a rate of 100 watts. If the light intensity falls off as the square of distance r, the power density observed from the light would be

$$\frac{100\,\mathrm{w}}{4\pi r^2}$$

Sample (sift) this function using your impulse at radii of 100 m, 1000 m, and 1000000 m to obtain the total observed intensity over a sphere at each distance.