Waves I | **EE241 Autumn 2005–2006**
Len Tyler | **Nov. 13, 2005**

Problem Set #7

Due Date: *Monday*, November 28, 2005. Submit in class, or outside Packard 331 before 4:30 PM.

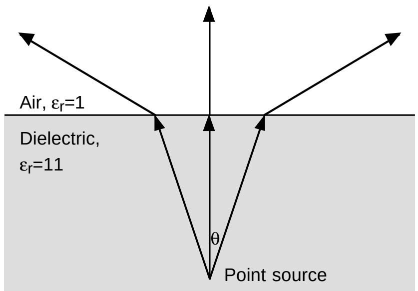
Reading Assignment:

"Reader"	Chapter 7,8,13
R W & VD	Sections 6.9-6.14

Problems:

1. Critical angle [30 points]

   Inside a light emitting diode (LED), light is emitted uniformly in all directions. At the surface of the LED, however, light is reflected from the dielectric-air interface. Use $\varepsilon_r = 11$ within the dielectric.

   (a) Find the maximum angle θ_{cr} at which you would expect the light from the LED to pass into the air. Assume the light comes from a uniform point source within the dielectric, and the surface of the dielectric is flat. [5 points]

   (b) Find the formula for the power transmission coefficients as a function of angle. Since there are two possible polarizations, you should have two equations. [10 points]

   (c) Estimate the fraction of power emitted from the surface. (This is sometimes known as the optical efficiency.) Assume power is evenly divided into TE and TM modes at the surface. Hint: Fraction of area subtended by angle θ_{cr} is

   $$\frac{2\pi \int_0^{\theta_{cr}} \sin\theta d\theta}{2\pi \int_0^{\pi} \sin\theta d\theta}$$

   [15 points]

2. Oblique Incidence Upon Multiple Dielectric Layers [20 points]

   An incident wave in medium 1 of permittivity ϵ_1 makes angle θ_1 with the normal. Find the proper length and permittivity of a medium 2 to form a "quarter-wave matching section" to a medium of permittivity ϵ_3. Consider both polarizations. [20 points]

3. Evanescent Waves [15 points]

   Sketch the electric field wave fronts of a TE plane wave undergoing total reflection at a plane interface, including the phase shifts. Sketch the incident, reflected, and evanescent wave fronts, as shown in Figures 3 and 23 in chapter 7 of the course reader. Use $\theta_i = 60°$, $\epsilon_1 = 4\epsilon_0$, $\epsilon_2 = \epsilon_0$, $\mu_1 = \mu_2 = \mu_0$. [15 points]

4. Shear Wave Propagation in a Crystal [20 points]

   A cubic crystal is aligned with the x, y, and z axes such that the non-zero elastic stiffness constants are:

$$\begin{aligned}
c_{xxxx} &= c_{yyyy} = c_{zzzz} = c_{11} \\
c_{xxyy} &= c_{yyxx} = c_{xxzz} = c_{zzxx} = c_{yyzz} = c_{zzyy} = c_{12} \\
c_{xyxy} &= c_{yxyx} = c_{xyyx} = c_{yxxy} \\
&= c_{xzxz} = c_{zxzx} = c_{xzzx} = c_{zxxz} \\
&= c_{zyzy} = c_{yzyz} = c_{zyyz} = c_{yzzy} \\
&= \frac{c_{44}}{2}
\end{aligned}$$

   (a) Write down the 6×6 stiffness matrix. [5 points]

   Now, suppose the following:

$$\sigma_{23} = \sin(\omega t - kz)$$

$$\epsilon_{23} = \frac{k^2}{2\rho\omega^2}\sin(\omega t - kz)$$

   and $c_{11} = 21.3 \times 10^{10}$ N/m^2, $c_{12} = 4.8 \times 10^{10}$ N/m^2, $c_{44} = 17.4 \times 10^{10}$ N/m^2, and $\rho = 4000$ kg/m^3.

   (b) Is this crystal isotropic? Why or why not? [2 points]

   (c) Find the direction of propagation and velocity of the wave. [5 points]

   (d) Why is this wave a shear wave? [3 points]

   (e) If the wave were a pressure wave traveling in the same direction, what would the velocity be? [5 points]

5. p- and s-wave Mode Conversion [40 points]

   A common technique for generating s-waves is to launch p-waves toward an air interface at the Brewster angle for that material. The reflected waves will be shear only.

   (a) What are the forms of the solutions for the p- and s-waves in this situation? [5 points]

   (b) Solve the boundary conditions to obtain a characteristic equation for the solution, relating λ, μ, α, γ_p, and γ_s. [15 points]

   (c) From the characteristic equation, find an equation relating θ_p to c_s/c_p showing when this will occur in an isotropic medium.

   Since $\sin\theta_s = \frac{c_s}{c_p}\sin\theta_p$, you may be pleased to recognize that

$$\cos\theta_s = \sqrt{1 - \left(\frac{c_s}{c_p}\right)^2 \sin^2\theta_p}$$

$$\sin^2\theta_s - \cos^2\theta_s = 2\left(\frac{c_s}{c_p}\right)^2 \sin^2\theta_p - 1$$

   [10 points]

   (d) For what range of c_s/c_p can this s-wave generation technique be used? Hint: You may want to use a math package such as Matlab, MathCAD, etc. [10 points]

6. Elastic-Rigid Frictionless Contact [30 points]

   An elastic medium (λ_0, μ_0, ρ_0) exists in the semi-infinite half-space $z > 0$ and borders a perfectly rigid material along the $z = 0$ plane. If an s-wave (not sh-wave) is incident from the elastic medium upon the boundary at an angle θ_i,

   (a) Sketch the wave vectors for the possible reflected waves. [5 points]

   (b) Write the solution forms for the total scalar and vector displacement potentials (ϕ, ψ). Then, state the displacements (ξ, η, ζ) using the potentials. [10 points]

   (c) Assuming a frictionless contact that does not separate, state the boundary conditions. [5 points]

   (d) Determine the reflection coefficients. [10 points]