EE 241: Waves I **Autumn 2005–2006**

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Problem Set #1

Due Date: October 7, 2005. Submit in class, or outside Packard Room 331 before 4:30 PM.

Reading Assignment:

"Reader" Chapters 1–3 Suggested Reading:

E&H Sections 1.1–1.5 (1.6–1.11 opt.)

Sections 5.1–5.3 (5.4 opt.)

Problems:

1. Operator notation [10 points]

   (a) Verify that $\phi(x,t)$ is a solution to by the wave equation by direct substitution. The one dimensional wave equation is given by $\phi_{xx} = \frac{1}{c^2}\ddot{\phi}$ and $\phi(x,t)$ is of the form

   $$\phi(x,t) = f_1(x - ct) + f_2(x + ct)$$

   . [5 points]

   (b) By use of the "chain rule" of differential calculus, establish the operator formulae

   $$2\frac{\partial}{\partial u} = \frac{\partial}{\partial x} - \frac{1}{c}\frac{\partial}{\partial t}$$

   $$2\frac{\partial}{\partial w} = \frac{\partial}{\partial x} + \frac{1}{c}\frac{\partial}{\partial t}$$

   where $u = x - ct$ and $w = x + ct$. [5 points]

2. Wave Equation [20 points]

   (a) Use the separation of variables method to find the general solutions for a three-dimensional wave equation (see below). Use k^2 for the separation constant, where $k^2 = k_x^2 + k_y^2 + k_z^2$. [15 points]

   $$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}$$

   (b) Interpret the physical meaning of your solution. Consider all possible values for k_x^2, k_y^2, and k_z^2. [5 points]

3. Initial Value Problem [30 points]

(a) A long string, for which the transverse velocity is c, is given a displacement specified by some function $\eta = f(x)$ that is localized near the center of the string. The string is released at $t = 0$ with zero initial velocity. Find the equations for the resulting traveling waves. Sketch the waves at several instants of time for $t > 0$ (Assume a simple shape for $f(x)$ to make your job easier).

Hint: You can solve this readily by finding two oppositely traveling waves that together satisfy the boundary condition at $t = 0$. [15 points]

(b) Now consider the situation where the string has not only an initial displacement but an initial velocity $\partial\eta/\partial t = g(x)$ at the time of release. Find the more general form of the resulting waves. [15 points]

4. Direction of Waves [20 points] Which of the following disturbances represents a travelling wave in one dimension (Give reason)? If its a travelling wave, what is the speed of the wave, and in which direction is it travelling? (Ignore the fact that some of these are unrealistic.)

   (a) $\eta = (3x - 4t)^2$

   (b) $\eta = x^2t^2$

   (c) $\eta = e^{-\alpha x}e^{j\omega t}$

   (d) $\eta = exp\left[-\alpha\,(2x - t)^2\right]$

   (e) $\eta = sin\,(4x + 3t) + sin\,(4x - 3t)$

5. Longitudinal waves [20 points]

   (a) What are longitudinal waves? [5 points]

   (b) Give three examples of longitudinal waves. [5 points]

   (c) A plane acoustic wave has incremental pressure

$$p(x,y,t) = 1.5sin\,[2\pi\,(1.2x - \beta y + 1000t + 2.41)]$$

   The propogation speed of the wave is 330 m/s and Air density is 1.29 Kg/m^3, and for a fixed "x" the wave appears to move in the positive "y" direction. Find β, the direction of propagation, wave number, frequency, angular frequency, period, peak pressure and average power carried by the wave.

6. System of springs and masses [10 points] With regard to the discussion of the system of springs and masses in Chapter 1, Fig. 2 of the "Course Reader":

   (a) How does the spring constant "K" depend on the length of the spring? Please be quantitative and state your reasoning. [5 points]

   (b) For the distributed system of uniform springs and masses, what are the units of $\mu * \kappa$? Does your answer balance the units of Eq. (15), p. 1-5 of the"Course Reader?" [5 points]