Lecture 9 - EE 179: Introduction to Communications - Spring 2009

Random Processes, WSS Processes, PSD, Filtering/Modulation, Gaussian Processes.

Lecture Outline

- **Random Processes**
- **Stationarity, Mean, and Autocorrelation**
- **Wide Sense Stationary (WSS) Processes**
- **Power Spectral Density**
- **White Noise.**
- **Filtering and Modulation**
- **Gaussian Processes**

1. **Random Processes**
   - A random process $X(t)$ is defined on a probability space $(\Omega, \mathcal{E}, P(\cdot))$.
   - A random process $X(t)$ is a function mapping from the sample space Ω to a set of functions.
   - Samples of $X(t)$ are joint random variables with joint CDF is $F_{X(t_0)X(t_1)\dots X(t_n)}(x_0,\dots,x_n) = P(X(t_0)\le x_0, X(t_1)\le x_1,\dots,X(t_n)\le x_n)$

2. **Stationarity**
   - A random process is (strictly) stationary if time shifts don't affect its probability.
   - So for all T and all sets of sample times $(t_0,\dots,t_n)$, $P(X(t_0)\le x_0, X(t_1)\le x_1,\dots,X(t_n)\le x_n) = P(X(t_0+T)\le x_0, X(t_1+T)\le x_1,\dots,X(t_n+T)\le x_n)$.

3. **Mean and Autocorrelation**
   - The mean of a random process is defined as $E[X(t)]$.
   - Stationary random processes have constant mean: $E[X(t)] = \mu_X$.
   - The autocorrelation of a random process is defined as $R_X(t_1,t_2) = E[X(t_1)X(t_2)]$.
   - For stationary random processes the autocorrelation depends only on time difference of the samples: $R_X(t_1,t_2) = R_X(\tau = t_2 - t_1)$.
   - Autocorrelation measures correlation between samples of the process taken at different times.

4. **Power Spectral Density (PSD)**
   - Defined only for WSS processes.
   - The PSD is the Fourier transform of the autocorrelation function: $R_X(\tau) \leftrightarrow S_X(f)$.
   - The expected power in $X(t)$ is the integral of its PSD: $E[X^2(t)]^H = \int S_X(f)df$.

5. **White Noise**

   - White noise is defined as a WSS random process with a flat PSD: $S_W(f) = N_0/2$
   - The autocorrelation of white noise is $R_W(\tau) = N_0/2\delta(\tau)$.
   - White noise is the "most random" of noise since it decorrelates instantaneously.

6. **Modulaton and Filtering**

   - For $X(t)$ a WSS process, passing $X(t)$ through a filter with impulse response $h(t)$ results in a WSS process $Y(t) = X(t) * h(t)$ with PSD $S_Y(f) = |H(f)|^2 S_X(f)$.
   - If we multiple a WSS process $X(t)$ by a cosine with uniformly distributed random phase, then $Y(t) = X(t)\cos(2\pi f_c t + \theta)$ is a WSS process with PSD $S_Y(f) = .25[S_X(f - f_c) + S_X(f + f_c)]$.

7. **Gaussian Random Process**

   - A random process $X(t)$ is Gaussian if for all times T and all functions $g(t)$ we get that $Y_g = \int_0^T g(t)X(t)dt$ is a Gaussian random variable.
   - Filtering a Gaussian process results in another Gaussian process.
   - Samples of a Gaussian random process are jointly Gaussian random variables.
   - Samples of a Gaussian random process that are uncorrelated are also independent.
   - WSS Gaussian processes are stationary.

Main Points:

- Random process $X(t)$ maps sample space Ω to a set of functions.
- Samples of $X(t)$ are joint RVs.
- A process is stationary if time shifts don't affect probability characteristics of the process.
- A WSS process has a constant mean and an autocorrelation that only depends on the time difference.
- PSD of a WSS process is the Fourier transform of its autocorrelation.
- White noise has a flat PSD.
- Modulation and filtering of random processes similar to deterministic case.
- Integrating a Gaussian process results in a Gaussian random variable.