

# Power Spectral Density and Autocorrelation.

## Lecture Outline

- Properties of PSD
- Filtering of Power Signals
- Modulation of Power Signals
- Autocorrelation of Power Signals
- PSD and Autocorrelation for Periodic Signals
- Random Signals

### 1. PSD Properties

- $S_x(f) \geq 0$
- $S_x(-f) = S_x(f)$

### 2. Filtering Power Signals

- Let  $x(t)$  be input to a filter with frequency response  $H(f)$ , where  $x(t)$  is a power signal with PSD  $S_x(f)$ .
- The filter output  $y(t)$  has PSD  $S_y(f) = |H(f)|^2 S_x(f)$ .

### 3. Modulation of Power Signals

- Let  $x(t)$  have a PSD  $S_x(f)$  that is bandlimited with bandwidth  $B$ .
- Then for  $z(t) = x(t) \cos(2\pi f_c t)$  with  $B \ll f_c$ ,  $S_z(f) = .25[S_x(f - f_c) + S_x(f + f_c)]$ .
- Similar to modulation property of Fourier transforms.

### 4. Autocorrelation of Power Signals

- Defined for real signals as  $R_x(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x(t)x(t - \tau)dt$ .
- As with ESD, autocorrelation is symmetric.
- Autocorrelation defines signal power:  $P = R_x(0)$ .
- As with ESD,  $|R_x(\tau)| \leq R_x(0)$ .
- The autocorrelation and PSD are Fourier Transform pairs:  $R_x(\tau) \Leftrightarrow S_x(f)$ .

### 5. PSD and Autocorrelation of Periodic Signals

- Periodic signals have a Fourier Transform  $x_p(t) \leftrightarrow \sum c_n \delta(f - n/T_0)$ .
- The PSD is obtained from the Fourier Transform  $S_{x_p}(f) = \sum |c_n|^2 \delta(f - n/T_0)$
- Since  $S_{x_p}(f)$  is discrete in the frequency domain, its inverse Fourier Transform, the autocorrelation function  $R_{x_p}(\tau)$ , is periodic in  $\tau$ .

## 6. Random Signals

- Random signals chosen from a set of possible realizations.
- Random signals characterized by their average PSD.
- Filtering and modulation of random signals has the same properties as for power signals.
- To study random processes, we need to develop a mathematical theory for analysis, which begins with probability theory.

### Main Points:

- PSD used to characterize modulation and filtering of power signals, which do not have Fourier transforms.
- Autocorrelation of a power signal measures its self-similarity.
- Autocorrelation and PSD are Fourier Transform pairs.
- Periodic signals have discrete PSDs and periodic autocorrelation functions.
- Random signals are also characterized by PSD and autocorrelation. Mathematical theory needed to study filtering and modulation of such signals.