

Signals and Information. System Block Diagram. Review of Fourier Series

Lecture Outline

- **Communication System Information Transfer**
- **Communication System Block Diagram**
- **Fourier Series Transform Pair**
- **Filtering Exponentials**
- **Fourier Series Properties**

1. **Communication Systems Transfer Information Electronically:**

- Communication systems convert information into a format appropriate for the communications channel, typically an electromagnetic wave (analog signal).
- Analog communication systems convert (modulate) analog information signals into modulated signals, which are also analog.
- Digital communication systems convert information in the form of bits into analog signals.
- Computers and digital devices generate information as bits. Analog information signals can be converted to bits by quantizing and digitizing.

2. **Types of Information: Analog Signals and Bits**

- Analog signals: voice, music, temperature readings, etc.
- Analog or digital signals: video and images
- Bits: text, computer data, etc.
- Analog signals can be converted into bits by sampling and quantizing.

3. **Communication System Block Diagram:**

- Source encoder: converts the source or input message into an analog signal or bits called the message signal.
- Transmitter: converts the message signal into a modulated signal in a format suitable for transmission over the channel.
- Channel: Introduces distortion and random noise. Linear channels have output $y(t) = x(t) * h(t) + n(t)$, where $x(t)$ is the transmitted (modulated) signal, $n(t)$ is the random noise, $h(t)$ is the channel impulse response, and $*$ denotes convolution.
- Receiver: Receiver extracts the original message signal or bits from the channel output signal.
- Source Decoder: Converts the message signal or bits back into the format of the original message.

4. Analog versus Digital Communication

- Analog communication systems transmit information in the form of analog waveforms (e.g. AM/FM radio, walkie talkies. TV). Digital communication systems transmit information in the form of bits (cell phones, computers, HDTV, XFM).
- Analog signals: varies over a continuous range. Most signals occurring in nature are analog.
- Binary signals: value takes on just two values (corresponding to “0”s and “1”s of bits).
- A *bit time* T is the amount of time required to send one bit. Data rate $R = 1/T$ bits per second (bps).
- Most modern communication systems are digital due to their robustness to noise and higher data rates.

5. Fourier Series Transform Pair:

- A periodic signal $x_p(t)$ with period $T_0 = 1/f_0$ has Fourier series representation

$$x_p(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t}.$$

- The Fourier series coefficients $\{c_n\}$ are given by $c_n = \frac{1}{T_0} \int_{T_0} x_p(t) e^{-j2\pi n f_0 t} dt$, where the integral is taken over any period of the signal.
- The Fourier series coefficients are the frequency components of the periodic signal.
- The transform exists if signal satisfies Dirichlet conditions over a period.

6. LTI Filtering of Exponential Functions

- Exponentials are eigenfunctions of LTI filters: if $x(t) = e^{j2\pi f t}$ is input into the filter, the filter output is $y(t) = H(f) e^{j2\pi f t}$. The complex constant $H(f)$ is called the frequency response of the filter at frequency f .
- If $x_p(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n t/T_0}$ is an LTI filter input, the filter output is $y(t) = \sum_{n=-\infty}^{\infty} c_n H(n/T_0) e^{j2\pi n t/T_0}$.

7. Periodic Square Wave Example:

- Fourier series coefficients are $c_n = A f_0 T \text{sinc}(n f_0 t)$, where $\text{sinc}(x) = \sin(\pi x)/(\pi x)$.
- Coefficients are symmetric ($c_n = c_{-n}^*$) with infinite frequency content.

8. Sinusoidal Fourier Series Representation:

$$x_p(t) = a_0 + 2 \sum_{n=1}^{\infty} [a_n \cos 2\pi n f_0 t + b_n \sin 2\pi n f_0 t],$$

where $a_n = \frac{1}{T_0} \int_{T_0} x_p(t) \cos 2\pi n f_0 t$, and $b_n = \frac{1}{T_0} \int_{T_0} x_p(t) \sin 2\pi n f_0 t$.

- Exponential to sinusoidal conversion: $a_n = .5(c_n + c_{-n})$, $b_n = -.5j(c_{-n} - c_n)$.

9. Properties of Fourier Series:

assume $x_p(t) \Leftrightarrow \{c_n\}$ and $v_p(t) \Leftrightarrow \{d_n\}$.

- *Linearity:* $Ax_p(t) + Bv_p(t) \Leftrightarrow \{Ac_n + Bd_n\}$.

- *Multiplication:* $x_p(t)v_p(t) \Leftrightarrow \{\sum_{k=-\infty}^{\infty} c_k d_{n-k}\}$.
- *Time Shifting:* $x_p(t - \tau) \Leftrightarrow \{e^{-j2\pi n f_0 \tau} c_n\}$.
- *Time Reversal:* $x_p(-t) \Leftrightarrow \{c_{-n}\}$.
- *Time Scaling:* $x_p(at) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 (at)}$
- *Conjugation:* $x_p^*(t) \Leftrightarrow \{c_{-n}^*\}$.
- *Parseval's Relation:*

$$\frac{1}{T_0} \int_{T_0} |x_p(t)|^2 dt = \sum_{n=-\infty}^{\infty} |c_n|^2.$$

Main Points:

- Communication systems modulate analog signals or bits for transmission over a communications channel.
- The building blocks of a communication system are meant to convert information into an electronic format for transmission, then convert it back to its original format after reception.
- Design goals of transmitter (modulator) and receiver (demodulator) are to mitigate impairments introduced by channel.
- Digital communication systems are more robust to noise and interference.
- Fourier series represents periodic signals in terms of exponential basis functions.
- Exponentials are eigenfunctions of LTI filters.