Lecture 17 - EE 179: Introduction to Communications - Spring 2009

Error Probability for Digital Modulation.

Lecture Outline

- **ASK and PSK Detection**
- **Noise in ASK/PSK**
- **Probability of Bit Error in ASK/PSK**
- **FSK Detection and Bit Error Probability**
- **Multilevel Modulation**

1. **ASK and PSK Detection**
   - Signal is multiplied by a carrier, integrated over a bit time, sampled, and passed through a decision device.
   - For ASK, when "1" transmitted, decision device input is $Y(nT_b) = \sqrt{E_b T_b/2}$, and when "0" transmitted, decision device input is $Y(nT_b) = 0$. Decision threshold is thus $.5(Y_0 + Y_1) = .25\sqrt{2E_b/T_b}$.
   - For PSK, when "1" transmitted, decision device input is $Y(nT_b) = \sqrt{E_b T_b/2}$, and when "0" transmitted, decision device input is $Y(nT_b) = -\sqrt{E_b T_b/2}$. Decision threshold is thus 0.
   - Noise immunity $\Delta N = .5(Y_1 - Y_0)$ is defined as half the distance between $Y(nT_b)$ corresponding to a "1" transmission and $Y(nT_b)$ corresponding to a "0" transmission.
   - Noise must exceed ΔN for decision device to make a bit error. For ASK, $\Delta N = .25\sqrt{2E_b/T_b T_b}$. For PSK, $\Delta N = .5\sqrt{2E_b/T_b}$.

2. **Noise in ASK/PSK**
   - An additive white Gaussian noise process in the channel introduces a noise term $N(nT_b)$ at the decision device input.
   - The distribution of $N(nT_b)$ is $\mathcal{N}(0, .25N_0T_b)$. This distribution is symmetric about zero, so $p(N(nT_b) > x) = p(N(nT_b) < -x)$.
   - The Q function or complementary error function describes this noise distribution. For N distributed as $\mathcal{N}(0, \sigma^2)$, $p(N > z) = Q(z) = .5\text{erfc}(\sqrt{.5}z) = \frac{1}{\sqrt{2\pi}}\int_z^\infty \exp[-.5x^2]dx$.

3. **Probability of Bit Error in ASK/PSK**
   - Assume "1" bits and "0" bits are equally likely.
   - For ASK, probability of bit error $P_b = .5p(N(nT_b) > .25\sqrt{2E_b/T_b T_b}) + .5p(p(N(nT_b) < -.25\sqrt{2E_b/T_b T_b}) = p(N(nT_b) > \sqrt{E_b/(8T_b)})$ by symmetry of Gaussian distribution.
   - Defining $Q(z)$ function as $p(x > z)$ for $x\ \mathcal{N}(0, 1)$, we get that for ASK, $P_b = Q(\sqrt{E_b/N_0})$, where E_b/N_0 is the bit energy to noise power spectral density ratio of ASK.
   - Similarly, for PSK, $P_b = Q(\sqrt{2E_b/N_0})$.

4. **FSK Detection and Bit Error Probability**

   - FSK Detection is done by splitting the signal into two branches, where each branch coherently demodulates the signal relative to its corresponding frequency $f_i, i = 1, 2$.
   - On each branch, the signal is multiplied by a carrier at frequency f_i, integrated over a bit time, and then sampled.
   - The sampled output on each branch is put into a comparator. The comparator outputs the bit corresponding to the branch with the bigger sampled input.
   - The comparator requires a minimum frequency separation of $|f_1 - f_2| = .5/T_b$. MSK modulation is FSK with this minimum separation.
   - Noise immunity for FSK is the same as for ASK ($\Delta N = \sqrt{E_b T_b/2}$).
   - For FSK, $P_b = Q(\sqrt{E_b/N_0})$.

5. **Multilevel Modulation**

   - In multilevel modulation m bits are encoded in a pulse of duration T_s.
   - Data rate is thus $R_b = m/T_s$ (in binary modulation $m = 1$).
   - The m bits are encoded in the amplitude A_n, phase θ_n, or frequency $f_n : \theta_n = 2\pi(f_n - f_c)t$ of the passband signal $s(t) = \sum_{n=-\infty}^{\infty} A_n \cos(2\pi f_c t + \theta_n)$, where A_n, θ_n, or f_n are constant over a symbol time T_s.
   - To send m bits per symbol, A_n, θ_n, or f_n must take on one of $M = 2^m$ possible values. Thus multilevel modulation is often referred to as M-ASK for multilevel ASK, MPSK for multilevel PSK, and MFSK for multilevel FSK, where the value of $M = 2^m$ may sometimes be used as well (e.g. 8PSK, which sends $m = \log_2(8) = 3$ bits per symbol).
   - 4PSK is an example of multilevel modulation where over a symbol time $s(t) = A_c \cos(2\pi f_c t + \theta_n)$ for $\theta_n = 0, \pi/2, \pi$, or $3\pi/2$.
   - MFSK is another example where over a symbol time $s(t) = A_c \cos(2\pi f_n t)$ for $f_n = f_1, f_2, f_3, \ldots, f_{2^m}$, where the minimum separation between frequencies is $.5/T_s$.
   - Demodulation of MPSK and MFSK is similar to the binary case.
   - Multilevel modulation increases data rate, but makes the signal more susceptible to noise.

Main Points:

- Decision device in ASK/PSK receiver uses threshold to determine which bit was sent.
- Bit errors occur when noise exceeds the noise immunity of the receiver, which is proportional to A_c and T_b.
- White Gaussian noise process causes a Gaussian noise term to be added to the decision device input.
- Bit error probability with white noise is a function of the symbol energy to noise spectral density ratio.
- FSK is less energy efficient than ASK/PSK but also less susceptible to amplitude variations.
- Multilevel modulation transmits multiple bits per symbol.