

# FM Modulation

## Lecture Outline

- Introduction to FM
- FM Bandwidth and Carson's Rule
- Spectral Analysis of FM
- Narrowband FM Modulation
- Wideband FM Modulation
- FM Detection

### 1. Introduction to Angle Modulation and FM

- In frequency modulation the carrier frequency or phase is varied relative to the information signal:  $s(t) = A_c \cos[\theta(t)]$ , where  $\theta(t)$  is a function of  $m(t)$ .
- FM signals are much less sensitive to amplitude variations in the channel. They are also less sensitive to signal reflections and refractions introduced by the channel.
- In standard FM,  $\theta(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau$ .
- The instantaneous frequency of an FM wave is  $f_i(t) = f_c + k_f m(t)$ .
- FM signals are categorized as narrowband FM or wideband FM, depending on the maximum magnitude of the information signal  $m(t)$ .

### 2. Spectral Analysis of FM

- Hard (impossible) for general signals  $m(t)$ .
- Set  $m(t) = A_m \cos(2\pi f_m t)$ , so  $B_m = f_m$ , then the FM modulated signal has spectrum

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t],$$

where  $\beta = \Delta f / f_m = k_f A_m / f_m$  and  $J_n(\beta)$  is a Bessel function of the first kind.

- The FM modulated signal has infinite bandwidth.
- For  $\beta \approx 1$ ,  $S(f)$  only has spectral content at  $f_c$  and  $f_c \pm f_m$ . This is called narrowband FM.
- Recall Carson's rule: FM signal bandwidth is  $B \approx 2\Delta f + 2B_m$ .  $\beta \approx 1$  implies  $\Delta f = k_f \max |m(t)| \ll B$ , which leads to narrowband FM.  $\Delta f \gg B$  leads to wideband FM.

### 3. Narrowband FM Modulation (NBFM)

- For NBFM  $\phi(t) = 2\pi k_f \int_0^t m(\tau) d\tau \ll 1$ .
- Can use the approximations  $\cos \phi(t) \approx 1$  and  $\sin \phi(t) \approx \phi(t)$ .
- Then  $s(t) \approx A_c \cos(2\pi f_c t) - A_c \sin(2\pi f_c t) \phi(t)$ , and this signal is easy to generate using a product modulator.

#### 4. Wideband FM Modulation (WBFM)

- Direct Method: use a VCO so that carrier frequency is proportional to input signal  $m(t)$ .
- Indirect Method: use a NBFM modulator followed by a nonlinear device and a bandpass filter.

#### 5. FM Detection

- Demodulator must extract  $m(t)$ , which is conveyed through the instantaneous frequency  $f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} = f_c + k_f m(t)$ ,
- Differentiation and Envelope Detection:  $s'(t) = A_c [2\pi f_c + k_f m(t)] \sin[2\pi f_c t + k_f \int_0^t m(\tau) d\tau]$ , so we extract the information signal by differentiating and then envelope detecting, as long as  $|k_f m(t)| < 2\pi f_c$ .
- In practice it is difficult to build a differentiator. A frequency discriminator approximates a differentiator within the bandwidth of interest.
- Zero crossing detector: approximates the instantaneous frequency  $f_i(t)$  by counting the number of zero crossings in a time interval  $T$ , where  $f_c^{-1} < T < B^{-1}$ .
- Phase-Locked Loop (PLL): the most common FM demodulator. Relies on a feedback principle to track the modulating signal  $m(t)$ .

#### Main Points:

- FM modulation encodes the information signal in carrier frequency instead of amplitude. It is much less sensitive to channel impairments (amplitude variations or signal reflections and refractions).
- Spectral analysis of FM difficult. For a simple cosine information signal, spectrum is discrete with infinite frequency content.
- NBFM is easier to analyze and is generated with a simple product modulator. WBFM is more complicated to analyze and to generate.
- In theory just need a differentiator and envelope detector for FM demodulation. Multiple methods used in practice, with PLL most common.