

Stationary and WSS, Mean/Autocorrelation Power Spectral Density, Modulation.

Lecture Outline

- Stationarity
- Mean and Autocorrelation
- Wide Sense Stationary (WSS) Processes
- Power Spectral Density
- White Noise
- Filtering and Modulation of Random Processes

1. Stationarity

- A random process is (strictly) stationary if time shifts don't affect its probability.
- So for all T and all sets of sample times (t_0, \dots, t_n) , $P(X(t_0) \leq x_0, X(t_1) \leq x_1, \dots, X(t_n) \leq x_n) = P(X(t_0 + T) \leq x_0, X(t_1 + T) \leq x_1, \dots, X(t_n + T) \leq x_n)$.

2. Mean and Autocorrelation

- The mean of a random process is defined as $E[X(t)]$.
- Stationary random processes have constant mean: $E[X(t)] = \mu_X$.
- The autocorrelation of a random process is defined as $R_X(t_1, t_2) = E[X(t_1)X(t_2)]$.
- For stationary random processes the autocorrelation depends only on time difference of the samples: $R_X(t_1, t_2) = R_X(\tau = t_2 - t_1)$.
- Autocorrelation measures correlation between samples of the process taken at different times.

3. Wide Sense Stationary Process (WSS)

- A process is WSS if its expected power is finite, $E[X^2(t)] < \infty$, its mean is constant, $E[X(t)] = \mu_X$, and its autocorrelation depends only on the time difference of the samples, $R_X(t_1, t_2) = R_X(\tau = t_2 - t_1)$.
- Intuitively, WSS processes are stationary in their first and second moments.
- Stationary processes are WSS but not vice versa.

4. Power Spectral Density (PSD)

- Defined only for WSS processes.
- The PSD is the Fourier transform of the autocorrelation function: $R_X(\tau) \leftrightarrow S_X(f)$.
- The expected power in $X(t)$ is the integral of its PSD: $E[X^2(t)] = \int S_X(f)df$.

5. White Noise

- White noise is defined as a WSS random process with a flat PSD: $S_W(f) = N_0/2$
- The autocorrelation of white noise is $R_W(\tau) = N_0/2\delta(\tau)$.
- White noise is the "most random" of noise since it decorrelates instantaneously.

6. Filtering and Modulation of Random Processes

- For $X(t)$ a WSS process, passing $X(t)$ through a filter with impulse response $h(t)$ results in a WSS process $Y(t) = X(t) * h(t)$ with PSD $S_Y(f) = |H(f)|^2 S_X(f)$.
- If we multiple a WSS process $X(t)$ by a cosine with uniformly distributed random phase, then $Y(t) = X(t) \cos(2\pi f_c t + \theta)$ is a WSS process with PSD $S_Y(f) = .25[S_X(f - f_c) + S_X(f + f_c)]$.

Main Points:

- Distribution of a stationary process is not affected by time shifts.
- The mean and autocorrelation of a random process generally depends on time, unless the process is stationary.
- A WSS process has a constant mean and an autocorrelation that only depends on the time difference.
- PSD of a WSS process is the Fourier transform of its autocorrelation.
- White noise has a flat PSD.
- Modulation of random processes similar to deterministic case.