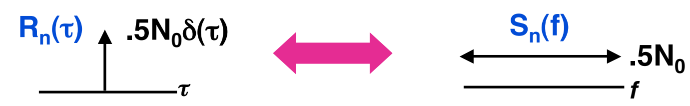
Lecture 16 Outline

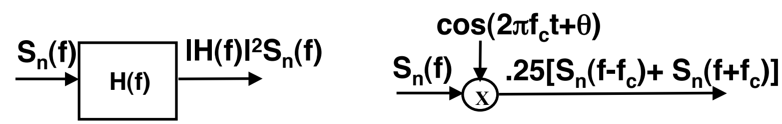
- Gaussian Processes
- Midterm Review
- Examples of noise in communications

Review of Last Lecture

- Power Spectral Density
- White Noise



- Filtering and Modulation



Gaussian Processes

- X(t) is a Gaussian process if

$$Y_g = \int_0^T g(t) X(t) dt$$

  is a Gaussian RV for any T and function g(t)

- Linear filtering a Gaussian process results in a Gaussian process
- Samples of a Gaussian process are jointly Gaussian random variables.
- Uncorrelated samples of a Gaussian process are independent.

Midterm Coverage:
Overview and Fourier (Chps 1-2)

- Overview of Communication Systems
  - Analog and digital signals
  - Bit times and data rates
  - Communication system block diagram
  - Shannon capacity
- Fourier Analysis
  - Periodic signals and Fourier series
  - Fourier transforms and their properties
  - Common Fourier transform pairs
  - Signal bandwidth
  - Sampling
  - Filtering
  - Ideal Filters
  - Equalization

Midterm Coverage: Power Signals, Probability (2.8-9, 8.1)

- Power Signals
  - Power spectral density
  - Autocorrelation
  - Modulation
  - Filtering
  - PSD and autocorrelation for periodic signals
  - Random signals characterized by PSD and autocorrelation
- Probability Theory
  - Probability space
  - Properties of probability
  - Conditional probability and Bayes Rule
  - Independent events
  - Partitions of the probability space

Midterm Coverage Random variables (Ch 8.2-8.4)

- Random Variables
  - Function mapping from Ω to real line
  - Discrete and continuous RVs
  - CDF and its properties
  - pdf and its properties
  - Mean, Moment, Variance, and Characteristic Functions
  - Functions of RVs
- Several random variables
  - Joint distributions
  - Joint moments, correlation and covariance
  - Independent random variables
- Gaussian random variable
  - pdf defined in terms of mean and variance
  - CDF not in closed form (in terms of error function or Q function)
  - Gaussian RVs that are uncorrelated are independent
  - Central Limit Theorem

Midterm Coverage: Random Processes (Ch 8.5-8.10)

- Random process X(t)
  - Defined on a probability space
  - Mapping from sample space to a set of functions
  - Samples of X(t) are joint RVs
  - Mean and autocorrelation
  - Stationarity
- WSS processes
  - $E[X^2(t)<\infty]$, mean constant, $R_X(t_1,t_2)=R_X(\tau=t_2-t_1)$
  - PSD is Fourier transform of $R_X(\tau)$
  - Expected power in X(t) is integral of its PSD
  - White noise has a flat PSD, decorrelates instantly
  - Modulation
  - Filtering
- Gaussian processes
  - Integrate to Gaussian RVs
  - Samples are jointly Gaussian
  - Filtering leads to another Gaussian process

Examples of noise in Communication Systems

- Gaussian processes: (thermal noise)
  - Filtering a Gaussian process yields a Gaussian process.
  - Sampling a Gaussian process yields jointly Gaussian RVs
  - If the autocovariance at the sample times is zero, the RVs are independent.
- The signal-to-noise power ratio (SNR) is obtained by integrating the PSD of the signal and integrating the PSD of the noise.
- In digital communications, the bit value is obtained by integrating the signal, and the probability of error by integrating Gaussian noise.

Main Course Ideas So Far

- Analog and digital signals convey information electronically. Key system parameters are data rate & performance.
- Fourier analysis is needed to study modulation, filtering, and channel effects in communication systems.
- PSD and autocorrelation needed to analyze power signals in communication systems.
- Probability theory characterizes random events.
- Random variables map from a probability space to the real line. Characterized by CDF, pdf, or characteristic function.
- Random processes map from a probability space to a set of functions (realizations). WSS processes characterized by PSD and autocorrelation, used to study filtering and modulation.
- White Gaussian noise is a common model for noise in communications. Determines SNR and probability of bit error.