Lecture 15 Outline

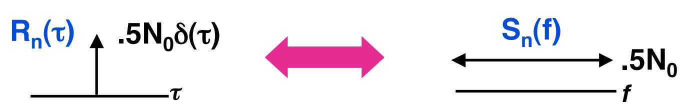
- Power Spectral Density
- White Noise
- Gaussian Random Processes

Review of Last Lecture

- Stationarity
  - Time shifts don't affect probability of process
- Mean of random process: E[X(t)]
  - Stationary process: $E[X(t)]=\mu_X$
- Autocorrelation of a random process:
  - Defined as $R_X(t_1,t_2)= E[X(t_1)X(t_2)]$
  - Stationary process: $R_X(t_1,t_2)=R_X(\tau=t_2-t_1)$
- Wide Sense Stationary Process (WSS)
  - $E[X(t)]=\mu_X$, $R_X(t_1,t_2)=R_X(\tau)$.
  - Properties of $R_X(\tau)$: $R_X(\tau)= R_X(-\tau)$, $|R_X(\tau)|\leq R_X(0)$
  - Autocorrelation measures how process decorrelates

Power Spectral Density (PSD)

- Defined only for WSS processes
- FT of autocorrelation function: $R_X(\tau)\Leftrightarrow S_X(f)$
- $E[X^2(t)]=\int S_X(f)df$
- White Noise: Flat PSD



- Decorrelates after infinitesimally small delay
- Good approximation in practice (thermal noise)
- Filtering white noise: introduces correlation

Gaussian Processes

- X(t) is a Gaussian process if

$$Y_g = \int_0^T g(t)X(t)dt$$

is a Gaussian RV for any T and function g(t)

- Linear filtering a Gaussian process results in a Gaussian process
- Samples of a Gaussian process are jointly Gaussian random variables.
- Uncorrelated samples of a Gaussian process are independent.

Main Points

- PSD of a WSS process is the FT of its autocorrelation
- White noise has flat PSD
- Gaussian random processes integrate to Gaussian RVs