Lecture 13 Outline

- Gaussian CDF: erf, erfc, and Q functions
- The Central Limit Theorem
- Random Processes

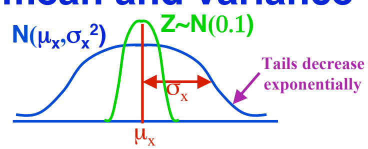
Review of Last Lecture

- Joint RVs: X and Y defined on (Ω,E,P)
  - Joint CDF $F_{XY}(x,y)=P(X\leq x,Y\leq y)$, joint pdf $f_{XY}(x,y)$.
  - Joint moments, correlation, and covariance.
  - Conditional pdf: $f_Y(y|X=x)= f_{XY}(x,y)/f_X(x)$.
  - Independent RVs: $f_{XY}(x,y)=f_X(x)f_Y(x)$.
- Sums of Independent RVs: Z=X+Y
  - $f_Z(z)=f_X(x)*f_Y(x) \Leftrightarrow \varphi_Z(\nu)=\varphi_X(\nu)\,\varphi_Y(\nu)$

Gaussian RVs and the CLT

- pdf defined in terms of mean and variance

$$f_X(x)=\frac{1}{\sqrt{2\pi}\sigma_X}e^{-[(x-\mu_X)^2/\sigma_X^2]}$$



- CDF defined by error function (erf(•))

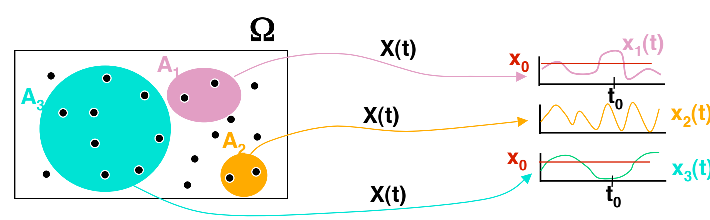
$$p(X\leq x)=F_X(x)=.5\left[1+\mathbf{erf}\left(\frac{x-\mu_X}{\sqrt{2}\sigma_X}\right)\right],\quad \mathbf{erfc}(u)=1-\mathbf{erf}(u),\quad \mathbf{Q}(x)=.5\mathbf{erfc}\left(x/\sqrt{2}\right)$$

Q(x)=P(X>x) for X~N(0,1).

- Central Limit Theorem: $X_1,\ldots,X_n$ i.i.d
  - Let $Y=\sum_i X_i$, $Z=(Y-\mu_Y)/\sigma_Y$
  - As $n\to\infty$, Z becomes Gaussian, $\mu_Y=0$, $\sigma_Y^2=1$.

Random Processes

- Defined on Probability Space (Ω,E,P)
  - Random process X maps Ω to a set of functions.



- Samples of X(t) are joint RVs:
  - $P(X(t_0)\leq x_0)=P(\cup A_i: x_i(t_0)\leq x_0)$
  - $P(X(t_0)\leq x_0, X(t_1)\leq x_1,\ldots,X(t_n)\leq x_n)=F_{X(t0)X(t1)\ldots X(tn)}(x_0,\ldots x_n)$

Main Points

- Probability of a Gaussian RV evaluated using erf, erfc, and/or Q functions
- Sums of i.i.d. shifted, normalized RVs converge to a $N(0,1)$ Gaussian RV.
- Random process X(t) maps Ω to a set of functions
- Samples of X(t) are joint RVs