Lecture 17: M-ary Digital Carier Modulation

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M-ary Digital Carrier Modulation

Lecture topics

- M-ary send many bits in a single symbol
- Many encoding schemes use different features of the pulses
  - Amplitude: PAM
  - Phase: PSK, QPSK, OQPSK
  - Frequency: FSK, MFSK, GMFSK
  - Many Frequencies: FDM, OFDM

Based on lecture notes from John Gill

M-ary Signalling

- There are many ways to send multiple bits of information with each pulse, or symbol.
- Send information by different amplitudes of a common pulse, or by sending different pulses

$$y(t) = \sum_k a_k p_k(t - kT_b)$$

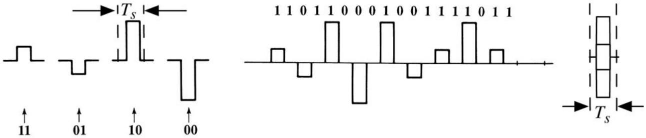
  where a_k is chosen from a set of more than two values (i.e., not just ± 1). Both $_k$ and $p_k(t)$ may be complex.
- Some methods use a common pulse $p(t)$ with different amplitudes
  - Amplitude shift keying (AKS) where a_k is real
  - Quadrature amplitude modulation (QAM) where a_k is complex
  - Phase shift keying (PSK) where a_k is complex, and unit magnitude
- Other methods use different waveforms and amplitudes
  - Frequency shift keying (FSK)
  - Orthogonal frequency domain multiplexing (OFDM)

Amplitude Shift Keying with ISDN

One widely used encoding of two bits into four levels is

$$a_k = \begin{cases} -3 & \text{message bits } 00 \\ -1 & \text{message bits } 01 \\ +1 & \text{message bits } 11 \\ +3 & \text{message bits } 10 \end{cases}$$

This is used in ISDN, a digital home and office network connection since replaced by DSL.

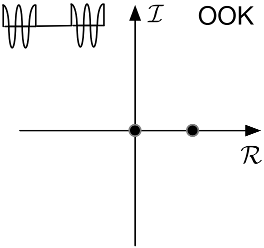


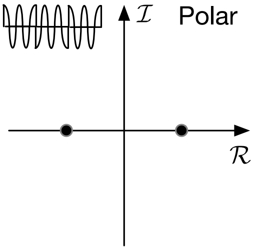
ASK used in

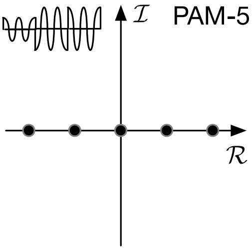
- Ethernet 100Base-T (PAM-3), 1000Base-T (PAM-5), 10GBase-T (PAM-16)
- GDDR6X memory (used in the RTX 3090) with PAM-4.
- ATSC Digital TV, (PAM-8)

Constellation Plots

- For all of the methods that use a common pulse $p(t)$ we can characterize them by plotting the coefficients a_k
- In general a_k can be complex, so the plot is in the complex domain.
- Some examples so far:



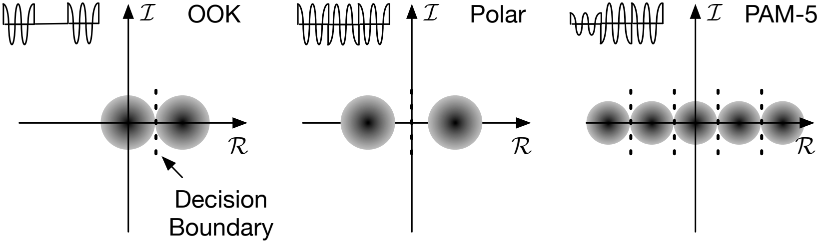




- These are all on the real axis.

Constellation Plots

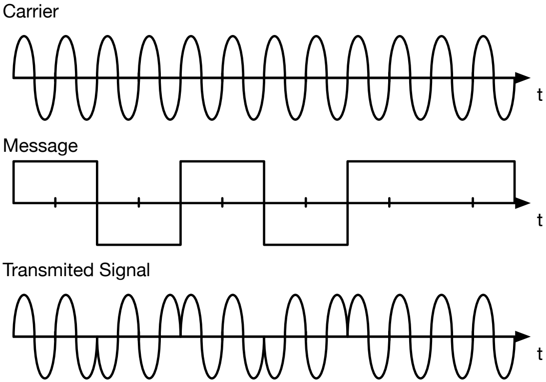
- Noise blurs the constellation causing decoding errors



- We want to pack more symbols in!
- Where should they go?

Binary Phase Shift Keying (PSK)

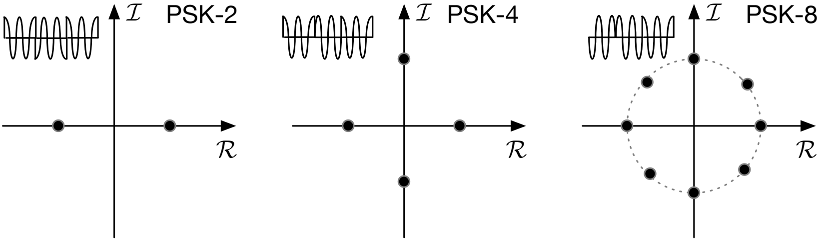
- Binary PSK is the same as polar ASK.



- Usually use an integer number of cycles of an offset frequency, and modulate that up to the carrier
- Phase changes at zero crossing to minimize bandwidth
- Detection generally must be synchronous, since the envelope is constant

M-ary Phase Shift Keying

- Binary phase shift keying is the same as polar signal
- Only two phase shifts are used, $\pm\pi$
- We can add more symbols by using additional phase shifts



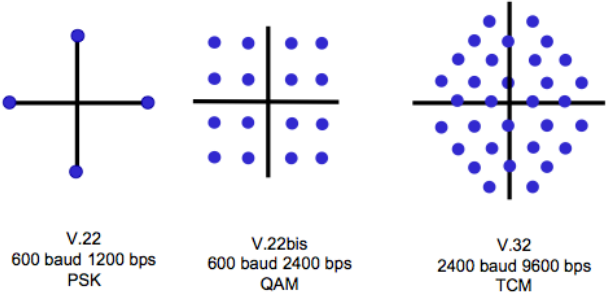
- The waveforms for M symbols are

$$p_k(t) = \cos(2\pi f_c t/T_b + 2\pi k/M)$$

- This is a coefficient $a_k = e^{j2\pi k/M}$ for the complex pulse $e^{j2\pi f_c t/Tb}$.
- Generally need a synchronous receiver

PSK Constellation Examples

These are all used in telephone modems:



- baud = symbol per second
- baud "rate" is proportional to bandwidth

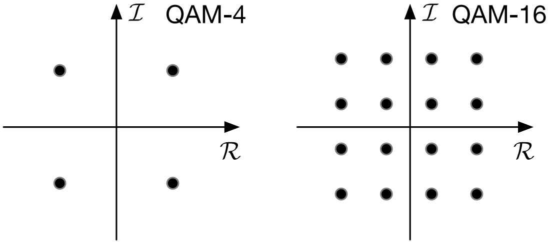
Quadrature AM (QAM)

- Another way to fill in the space is to use linear combinations of sines and cosines

$$p_k(t) = a_k \cos(2\pi f_c t/T_b) + b_k \sin(2\pi f_c t/T_b)$$

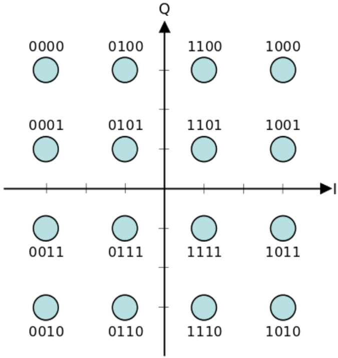
  The cosine term is along the real axis, and the sine term along the imaginary axis

- The constellations look like

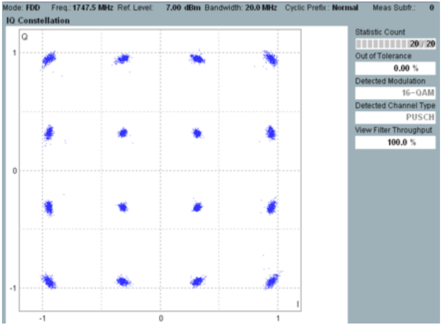


- Here the symbols are spaced rectilinearly and evenly, many other options
- M-bits / symbol
- Common for cable TV, where QAM-64 or QAM-256 are used.

QAM Constellation Measurement



QAM-16



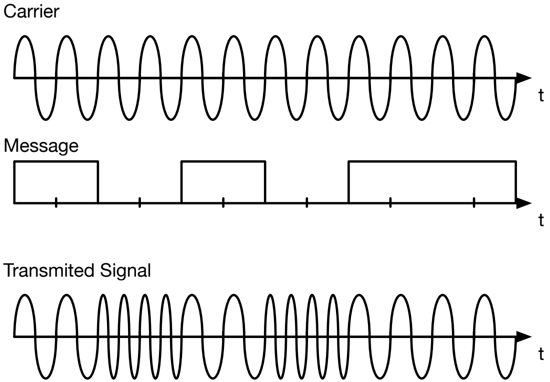
Experimental Measurement

- Note the ordering. Adjacent symbols differ by only one bit. This is called a Gray code.
- Errors due to noise will mostly be between adjacent symbols
- These will result in single bit errors that are easily correctable (next class).

Illustrations from Wikipedia

Binary Frequency Shift Keying FSK)

- Binary FSK uses two frequencies for 1 and zero.



- Usually integer numbers of cycles of each offset frequency, so that they are orthogonal
- Easy to receive, can be done with filters and an envelope detector (see this week's lab!). Does not need to be synchronous.

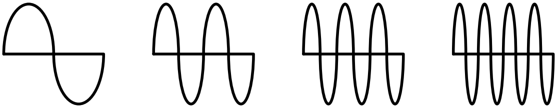
M-ary Frequency Shift Keying (MFSK)

- For M-ary FSK we use several frequencies

$$p_k(t) = \cos(2\pi(f_c + k\Delta f)t)$$

  where Δf is the spacing in frequency.

- Decoding is easy if $(\Delta f)T_b = 1$, so that we have integer numbers of cycles of the cosine (or sine).



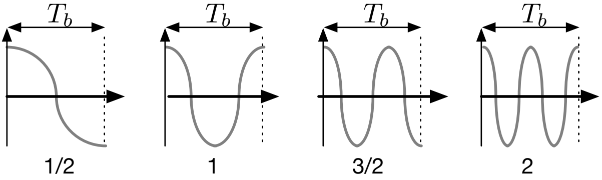
  This is a modulation index $m = 1$, meaning the frequencies are multiples of one cycle.

- These are all orthogonal

- The receiver can use an array of filters, one for each frequency, followed by enveloped detectors, just as in the binary case.

Minimum Shift Keying (MSK)

- If we want to get the most bits/symbol across a channel for FSK, we'd like the frequencies to be as close together as possible
- The minimum duration orthogonal signals are multiples of *half* a cycle



  These are all orthogonal. It only works because we are limiting the waveforms to only cosines.

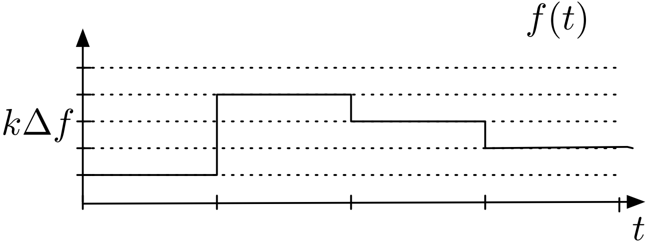
- These FSK frequencies are Minimum Shift Keying, MSK
- The modulation index is $m = 0.5$, and all the frequencies are multiples of that for one half cycle
- At this point the receiver has to be more sophisticated, simple filters and envelope detectors won't suffice.

Gaussian Minimum Shift Keying (GMSK)

- FSK can be thought of as a time varying frequency waveform

$$y(t) = \cos(2\pi(f_c + f(t))t)$$

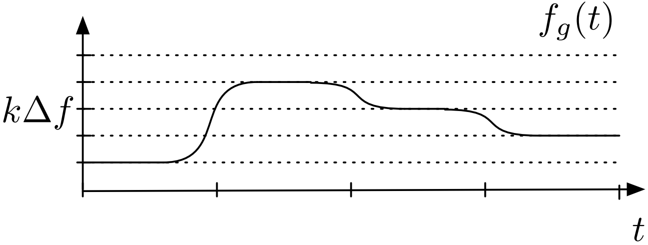
where $f(t)$ looks like this:



- The abrupt frequency changes cause sidelobes, and interfere with adjacent channels

Gaussian Minimum Shift Keying (GMSK)

- To minimize this, the $f(t)$ waveform is first convolved with a Gaussian waveform to smooth it



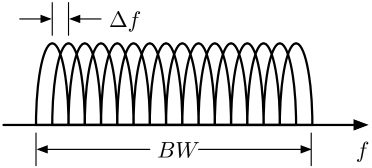
- If this is used with Minimum Shift Keying, the results is Gaussian Minimum Shift Keying or GMSK
- This is used in GSM cell telephones, with a modulation index of $m = 0.3$, even less than MSK!
- The results is ISI, and mitigation strategies and error correction are required to make it work.

Orthogonal Frequency Domain Multiplexing, OFDM

- So far we have been trying to get as many symbols per second through the channel, each encoded for as many bits as possible
- Another approach is to go for very large number of frequencies per symbol, so that the symbol rate can be much lower
- If we have a sequence c_k of bits or QAM encoded symbols, we can modulate (FSK encode) one sample as

$$d_k(t) = c_k e^{j2\pi k \Delta f t}$$

  Each sample c_k is transmitted at its own FSK frequency $k\Delta f$
- Each frequency we can think of as an independent subchannel



OFDM

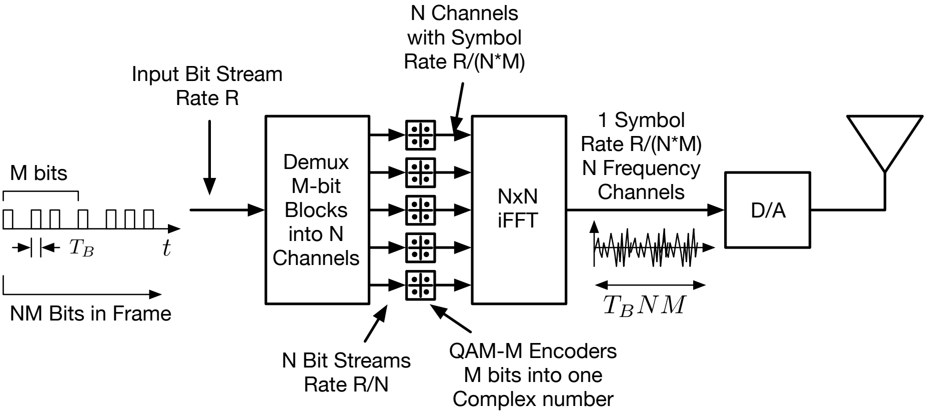
- We can send data on all of them at once by adding the signals up
- If we combine all of the samples in an interval (frame) we get

$$d(t) = \sum_{k=0}^{N-1} c_k e^{j2\pi k \Delta f t}$$

- If there are N samples, then there are N output FSK frequencies
- In practice $d(t)$ is sampled, and this operation is computed with an inverse FFT.
- Very effectively uses the entire available spectrum
- Different channels will have different noise and propagation properties
- We can equalize each independently, and allocate more or less bits to each

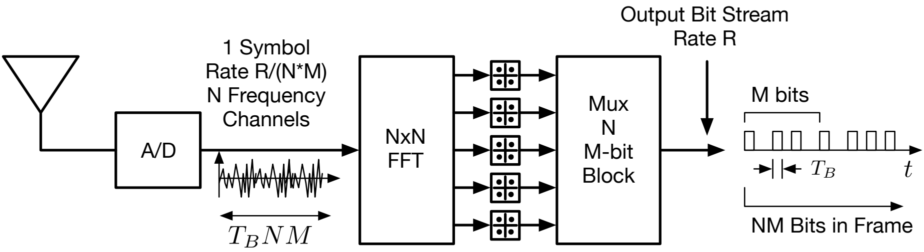
OFDM Encoder

The transmitter looks like this



OFDM Decoder

The receiver basically just inverts the operations of the transmitter



OFDM

- OFDM has many practical advantages
  - The many different frequency channels are resistant to channel variations
  - The much lower symbol rate makes timing and pulse shaping much easier
- The number of frequencies can be anywhere from 64 to 8k or more
- The constellation encoder may be anything from BPSK through QAM-256
- That can be a lot of bits per symbol!
- OFDM is widely used
  - Digital Satelite TV : DVB-T, and others
  - WiFi : 802.11a/g/n and WiMAX
  - Digital Radio : Digital Radio Modiale (DRM)
  - ADSL

Next Couple of Classes

- Friday : Projects topics
- Next Week : Off
- Monday : Error correction, parity bits, CRC codes
- Wednesday : Spread spectrum, Radar, GPS, CDMA
- Friday : Work on projects