

Lecture 10: Angle Modulation with Wideband FM

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Angle Modulation: Wideband FM

Lecture topics

- ▶ FM Modulation
- ▶ FM Demodulation
- ▶ Spectral pre-emphasis and de-emphasis to improve SNR

Based on lecture notes from John Gill

US Broadcast FM

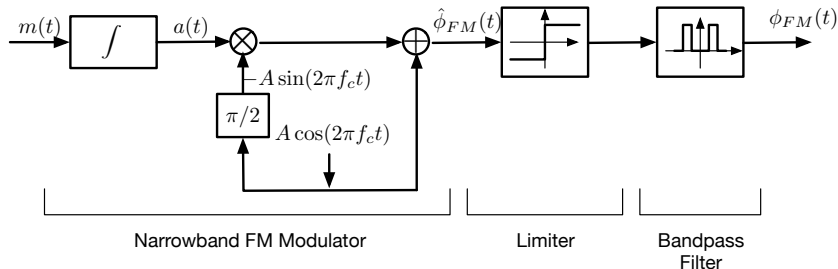
- ▶ US FM bandwidth specifications:
 - ▶ Frequency range: 88.0 – 108.0 MHz
 - ▶ Channel width: 200 kHz (100 channels)
 - ▶ Channel center frequencies: 88.1, 88.3, ..., 107.9
 - ▶ Frequency deviation: ± 75 kHz
- ▶ Signal bandwidth: high-fidelity audio requires ± 15 kHz, so bandwidth is available for other applications:
 - ▶ Stereo $L - R$ channel
 - ▶ Extra channels
 - ▶ Radio data service (RDS)

NBFM Modulation

For narrowband signals, $|k_f a(t)| \ll 1$ and $|k_p m(t)| \ll 1$,

$$\hat{\phi}_{\text{NBFM}} \approx A(\cos(2\pi f_c t) - k_f a(t) \sin(2\pi f_c t))$$

We can use a DSB-SC modulator with a phase shifter, followed by a limiter and a bandpass filter.



Armstrong's indirect method

Armstrong wanted to generate WBFM using NBFM and frequency multipliers. Suppose we have a nonlinear device with the response

$$y(t) = a_0 + a_1x(t) + a_2x^2(t) + \cdots + a_nx^n(t)$$

Let $a(t) = \int_{-\infty}^t m(u) du$ and

$$x(t) = A \cos(2\pi f_c t + k_f a(t))$$

Then

$$y(t) = a_0 + a_1 \cos(2\pi f_c t + k_f a(t)) + \cdots + a_n (\cos(2\pi f_c t + k_f a(t)))^n$$

The last term has a component at nf_c with a modulation of $nk_f a(t)$. It is n times higher in frequency and wider in spectrum.

We can select this with a bandpass filter.

FM Demodulation

There are many ways to demodulation the WBFM signal. Most use differentiation one way or another.

- ▶ Differentiator
- ▶ Slope detection
- ▶ Frequency-selective filter
 - ▶ RC high-pass filter

$$H(f) = \frac{j2\pi RCf}{1 + j2\pi RCf} \approx j2\pi RCf \quad (2\pi RC \ll 1)$$

- ▶ RLC circuit with carrier frequency $\omega_c < \omega_0 = 1/\sqrt{LC}$
- ▶ Zero-crossing detectors
- ▶ Phase-locked loop

Derivative Theorem for Fourier Transform

If $G(f)$ is the Fourier transform of $g(t)$, then

$$\frac{dg(t)}{dt} \Leftrightarrow j2\pi f G(f)$$

and

$$\int_{-\infty}^t g(\tau) d\tau \Leftrightarrow \frac{G(f)}{j2\pi f} + \frac{1}{2}G(0)\delta(f)$$

“Proof”:

$$\begin{aligned}\frac{dg(t)}{dt} &= \frac{d}{dt} \int_{-\infty}^{\infty} G(f) e^{j2\pi ft} dt \\ &= \int_{-\infty}^{\infty} \frac{d}{dt} G(f) e^{j2\pi ft} dt = \int_{-\infty}^{\infty} j2\pi f G(f) e^{j2\pi ft} dt\end{aligned}$$

By the Fourier inversion theorem, $j2\pi f G(f)$ is transform of $g'(t)$.

FM Detection by Differentiation

The complex FM signal is

$$\begin{aligned}\hat{\varphi}_{\text{FM}}(t) &= Ae^{j(2\pi f_c t + k_f a(t))} \\ &= Ae^{jk_f a(t)} e^{j2\pi f_c t}\end{aligned}$$

The transmitted signal is just the real part, $\varphi_{\text{FM}}(t) = \text{Re}(\hat{\varphi}_{\text{FM}}(t))$.

First we demodulate the signal to baseband, which eliminates the carrier

$$\hat{\varphi}_{\text{BB}}(t) = Ae^{jk_f a(t)}$$

Then we differentiate

$$\begin{aligned}\dot{\hat{\varphi}}_{\text{BB}}(t) &= \frac{d}{dt} \left(Ae^{j(k_f a(t))} \right) \\ &= A(jk_f m(t))e^{j(k_f a(t))}\end{aligned}$$

Note that for this to work A must really be constant.

Extracting the Message with and SDR

We can then extract the FM signal by cancelling out the phase with the original signal

$$\begin{aligned}\dot{\hat{\phi}}_{BB}(t)\hat{\phi}_{BB}^*(t) &= A(jk_fm(t))e^{j(k_f a(t))} A e^{-j(k_f a(t))} \\ &= A^2(jk_fm(t))\end{aligned}$$

This is the message with a scale factor A^2jk_f . This is what we will do in the lab.

However, there is a simpler way if we don't demodulate down to baseband.

- ▶ The message is in the envelope after differentiation, so we could use an envelope detector just like we do in AM.
- ▶ However, if we demodulate to baseband we same issue as with DSC-AM, with sign inversions, since $m(t)$ is both positive and negative.
- ▶ The solution is to demodulate to an intermediate frequency, and then envelope detect

WBFM Envelope Detection

After demodulation to an intermediate frequency f_i ,

$$\hat{\varphi}_{IF}(t) = Ae^{jk_f a(t)} e^{j2\pi f_i t}$$

Then when we differentiate we get

$$\begin{aligned}\dot{\hat{\varphi}}_{IF}(t) &= \frac{d}{dt} \left(Ae^{j(k_f a(t))} e^{j2\pi f_i t} \right) \\ &= Aj(2\pi f_i + k_f m(t)) e^{j(k_f a(t))} e^{j2\pi f_i t}\end{aligned}$$

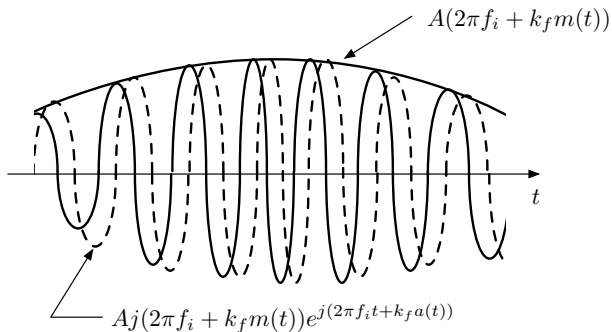
As long as $2\pi f_i > k_f m(t)$ the envelope will always be positive, and we can just take the magnitude of the signal, just like we did with AM.

Then

$$\left| \dot{\hat{\varphi}}_{IF}(t) \right| = A(2\pi f_i + k_f m(t))$$

The DC term due to f_i is then filtered out by a highpass filter. This is a more common way to demodulate WBFM.

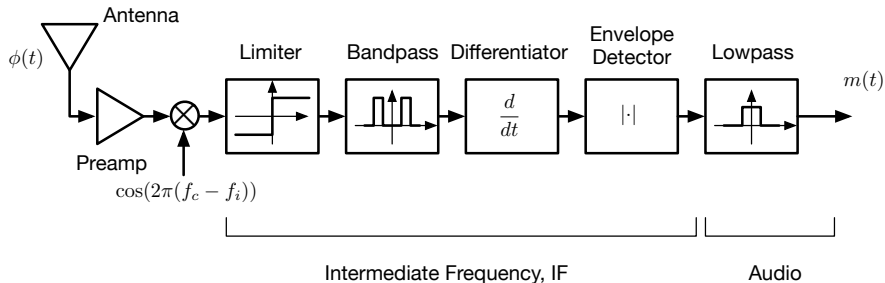
FM Demodulator and Differentiator



The envelope is increases with frequency, and is always positive.
The same envelope detector that we used for AM will work here.

WBFM Envelope Detection

- ▶ The block diagram now looks like



- ▶ By demodulating to an IF, we can use an envelope detector

Advantages of FM

FM is less susceptible to amplifier nonlinearities. If input is

$$x(t) = A \cos(\omega_c t + \psi(t))$$

and the output is

$$\begin{aligned} y(t) &= a_0 + a_1 x(t) + a_2 x^2(t) + \dots \\ &= c_0 + c_1 \cos(\omega_c t + \psi(t)) + c_2 \cos(2\omega_c t + 2\psi(t)) + \dots \end{aligned}$$

The extra terms have spectrum outside the carrier signal band. They will be blocked by bandpass filter.

Nonlinearities in AM cause signal distortion. For $y(t) = ax(t) + bx^3(t)$,

$$\begin{aligned} y(t) &= am(t) \cos \omega_c t + bm^3(t) \cos^3 \omega_c t \\ &= (am(t) + \frac{3}{4}bm^3(t)) \cos \omega_c t + \frac{1}{4}b \cos 3\omega_c t \end{aligned}$$

FM is preferred for high power applications, such as microwave relay towers.

Advantages of FM (cont.)

FM can adjust to rapid fading (change of amplitude) using AGC (automatic gain control)

FM is less vulnerable to signal interference from adjacent channels.

Suppose interference is $I \cos(\omega_c + \omega)t$. Then received signal is

$$\begin{aligned}r(t) &= A \cos \omega_c t + I \cos(\omega_c t + \omega)t \\ &= (A + I \cos \omega t) \cos \omega_c t - I \sin \omega t \sin \omega_c t \\ &= E_r(t) \cos(\omega_c t + \psi(t))\end{aligned}$$

where

$$\psi(t) = \tan^{-1} \left(\frac{I \sin \omega t}{A + I \cos \omega t} \right) \approx \frac{I}{A} \sin \omega t \quad (I \ll A)$$

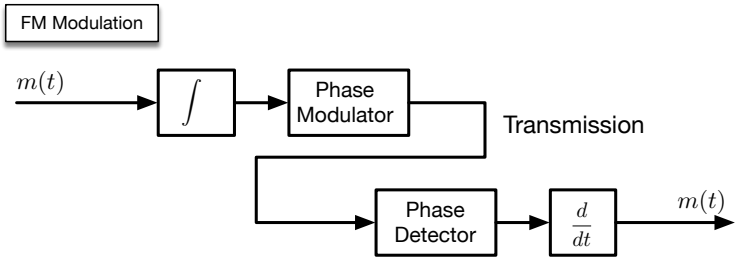
The output of an ideal frequency modulator is $\dot{\psi}(t)$ for FM is

$$y_d(t) = \frac{I\omega}{A} \cos \omega t,$$

which is inversely proportional to amplitude A .

Noise and FM

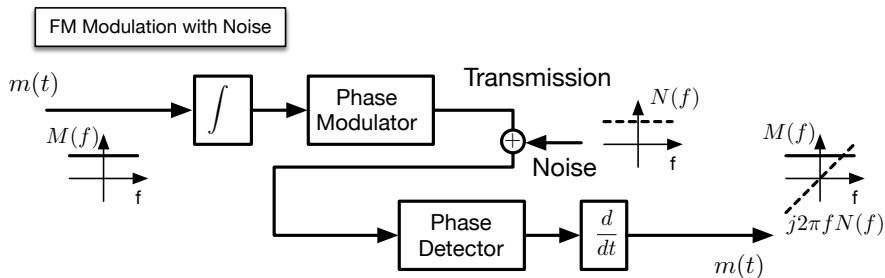
- ▶ The FM modulator and demodulator looks like



- ▶ The integration and differentiation operations are inverses
- ▶ The issue is what happens when noise is added during transmission.

Differentiation Accentuates Noise

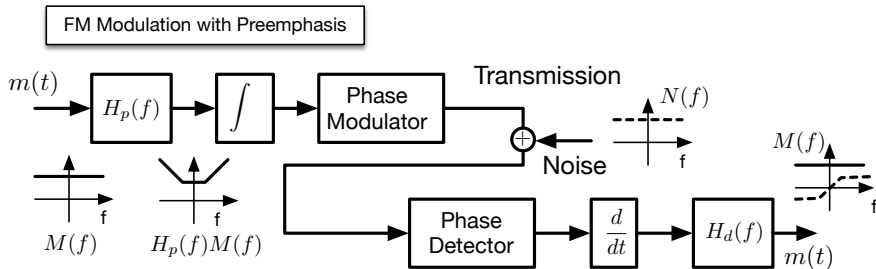
- ▶ Noise has a flat spectrum, and is added during the transmission



- ▶ After the differentiator, noise has a highpass characteristic, while the message is flat spectrally
- ▶ We would like the signal to noise ratio (SNR) to be flat

Preemphasis

- ▶ Solution is to add an preemphasis filter $H_p(f)$ to amplify the high frequency signal

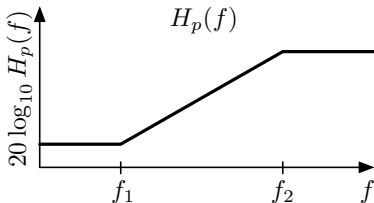
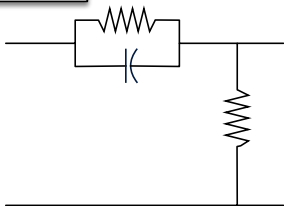


- ▶ A deemphasis filter $H_d(f)$ compensates the spectrum so that $H_p(f)H_d(f) = 1$.
- ▶ The noise and signal spectra are now flat at the output.
- ▶ The preemphasis is approximately a differentiator! Broadcast FM is effectively a PM system!

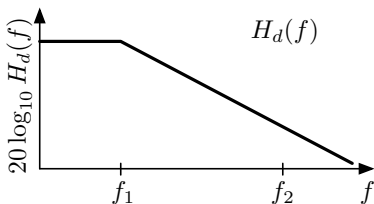
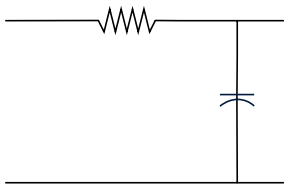
FM Preemphasis and Deemphasis

Pre-emphasis: RLC high pass filter. De-emphasis: RC low pass filter.

Preemphasis



Deemphasis



FM Preemphasis and Deemphasis (cont.)

The linear preemphasis range is $f_1 = 2.1$ kHz to $f_2 = 30$ kHz. The preemphasis filter has transfer function

$$H_p(f) = \frac{f_2}{f_1} \frac{f_1 + j2\pi f}{f_2 + j2\pi f}$$

If $f \ll f_1$ then $H_p(f) \approx 1$.

If $f_1 \ll f \ll f_2$ then

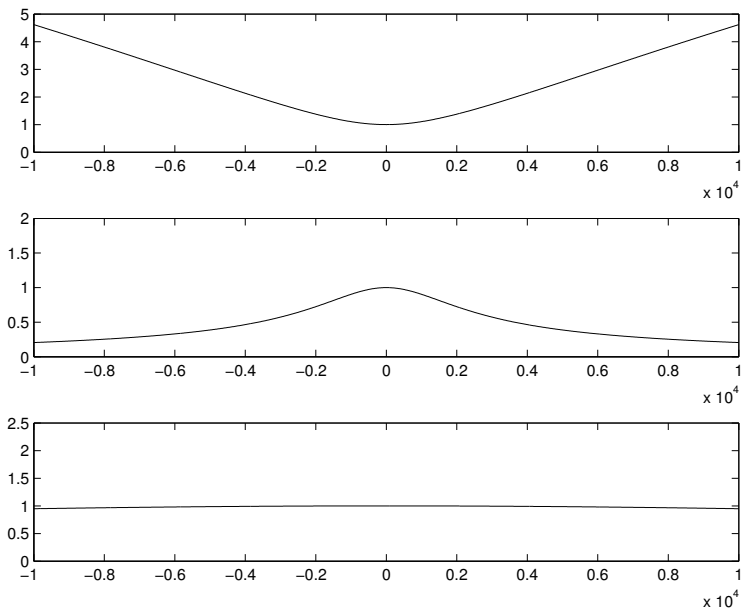
$$H_p(f) \approx \frac{j2\pi f}{f_1}$$

which is a differentiator!

The corresponding deemphasis filter has transfer function.

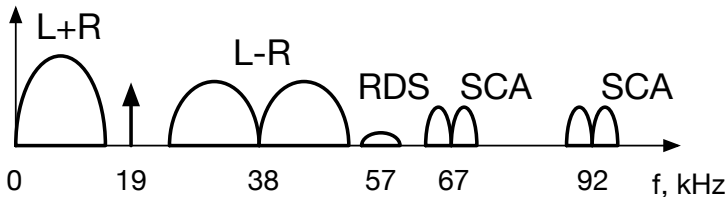
$$H_d(f) = \frac{f_1}{j2\pi f + f_1} \approx \frac{1}{H_p(f)}$$

FM Preemphasis and Deemphasis Filters



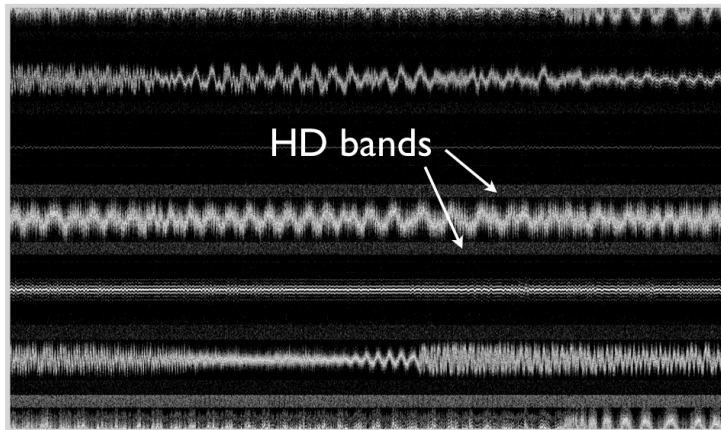
Broadcast FM Signal

- ▶ So far we've just been talking about WBFM. The broadcast signal has a lot more in it than the single channel we have so far described.
- ▶ Broadcast FM signal is stereo, and has many components
 - ▶ Left+Right in the middle of the spectrum
 - ▶ Left-Right offset by 38 kHz
 - ▶ 19 kHz pilot tone
 - ▶ Radio Data System (RDS) digital signal at 57 kHz
 - ▶ Two Subsidiary Communications Authorization (SCA) signals at 67 and 92 kHz
- ▶ It may also have up to four High Definition (HD) bands



Broadcast FM Signal Spectrogram

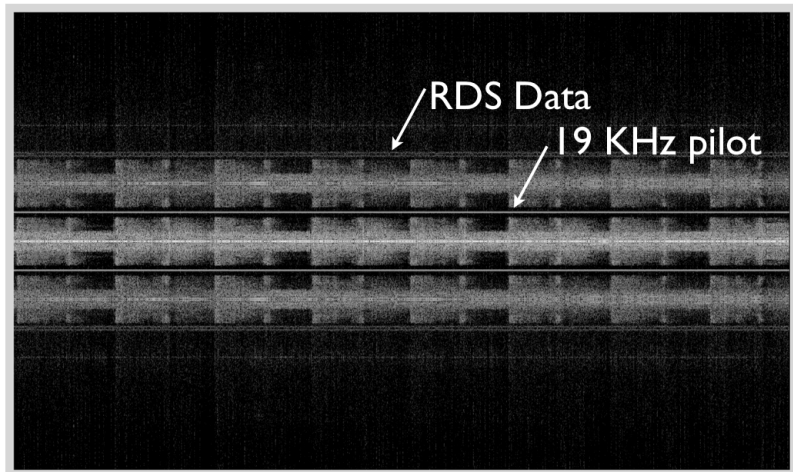
This is 2 MHz of the broadcast FM band centered at 104.5 MHz.



104.5 MHz

Broadcast FM Signal Demodulated

After decimating and demodulating the previous signal, we get the demodulated FM signal.



L-R
L+R
L-R

+/- 100 kHz Centered on 104.5 MHz

Next time

- ▶ Wednesday : Sampling, analog multiplexing methods
- ▶ Friday : Lab on Stereo Encoding Methods