

**Homework 7: Due Friday June 5 at 5 pm (no late HWs).**

1. (25 points) This problem illustrates design choices and limitations for certain FM detector designs. Consider an FM system where the modulated signal is  $s(t) = 10 \cos \left[ 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right]$  for a carrier frequency of 100 MHz. Assume the modulating signal  $m(t) = 10 \cos(2\pi f_m t)$  for  $f_m = 3$  KHz.

(a) What is the maximum value of  $k_f$  such that you can demodulate  $s(t)$  using an ideal differentiator followed by an envelope detector?

*For the remainder of the problem assume that  $k_f = 10$ .*

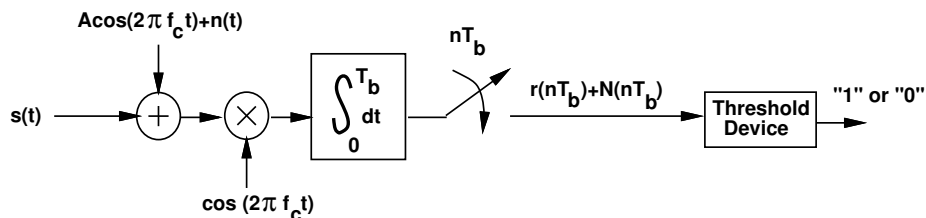
(b) What is the approximate bandwidth of  $s(t)$ ? Is this NBFM or WBFM?

(c) Find the instantaneous frequency  $f_i(t)$  of  $s(t)$ . What are the maximum and minimum values of  $f_i(t)$ ?

(d) Suppose you demodulate  $s(t)$  using an ideal differentiator followed by an envelope detector. Assume a standard envelope detector as shown in Figure 3.9a of the book, where the capacitor has capacitance  $C = 10^{-9}$ F. Propose values for the source resistance  $R_s$  and load resistance  $R_l$  such that the output of the envelope detector is approximately equal to  $c_1 + c_2 m(t)$  for some constants  $c_1$  and  $c_2$ . Is it possible to use this detection method if  $f_c \approx f_m$  (why or why not)?

2. (25 points) Consider a BPSK receiver as in Figure 10.7 where the demodulator has a phase offset of  $\phi$  relative to the transmitted signal  $s(t) = \pm A_c m(t) \cos(2\pi f_c t)$ . Assume AWGN  $n(t)$  with mean zero and PSD  $N_0/2$ . Find the decision device input  $Y$  corresponding to sending a “1” bit and a “0” bit as a function of  $\phi$ . Then determine the appropriate threshold level in the decision device given these values. Finally, find the probability of bit error as a function of  $\phi$ ,  $A_c$ ,  $T_b$ ,  $N_0$  and as a function of  $\phi$  and  $E_b/N_0$ .

3. (25 points) This problem illustrates the impact on bit error probability of a deterministic interference signal in BPSK. Consider the BPSK system shown below. The signal  $s(t) = m(t) \cos(2\pi f_c t)$  is a BPSK modulated signal where  $m(t)$  is a baseband polar modulated signal with amplitude  $A_c$ . The channel introduces both additive white noise  $n(t)$  with PSD  $N_0/2$  and a deterministic interference term  $A \cos(2\pi f_c t)$  at the carrier frequency. The input to the threshold device is  $r(nT_b) + N(nT_b)$  where  $N(nT_b)$  is based on  $n(t)$  and  $r(nT_b)$  depends on both  $s(t)$  and the interference signal  $A \cos(2\pi f_c t)$ . The threshold device decides that a “1” was sent if its input is greater than zero, otherwise it decides that a “0” was sent. Assume  $f_c T_b \gg 1$ .

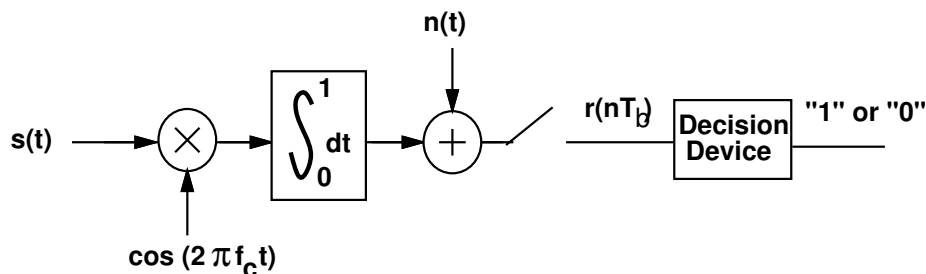


(a) Find the statistics of  $N(nT_b)$ .

- (b) Find  $r(nT_b)$  when a “1” is sent and when a “0” is sent.
- (c) Find  $p(\text{detect “1”}|\text{send “0”})$  in terms of the Q function ( $Q(x) = .5\text{erfc}(x/\sqrt{2})$ ),  $A_c$ ,  $A$ ,  $T_b$ , and  $N_0$ .
- (d) Find  $p(\text{detect “0”}|\text{send “1”})$  in terms of the Q function,  $A_c$ ,  $A$ ,  $T_b$ , and  $N_0$ .
- (e) Find the probability of bit error,  $P_b$ , assuming equally likely bits ( $p(\text{send “1”}) = p(\text{send “0”}) = .5$ ).
- (f) Evaluate  $P_b$  assuming  $A_c = 4$ ,  $A = 1$ , and  $T_b/N_0 = 1$ . Compare this with  $P_b$  assuming  $A_c = 4$ ,  $A = 0$ , and  $T_b/N_0 = 1$  (i.e. the case of no interference).
4. (25 points) Consider the FSK demodulator in Figure 10.12, where  $s(t) = A_c \cos(2\pi f_i t)$  for  $f_i = f_1$  (“1” sent) or  $f_i = f_2$  (“0” sent). So for equally likely bits  $E_b = .5A_c^2 T_b$ . The decision device outputs a “1” if  $Y_1 - Y_2 > T$  and a “0” if  $Y_1 - Y_2 \leq T$  where  $T$  is a threshold level. Assume that  $f_1$  and  $f_2$  are on the order of 1 GHz.
- (a) Assume  $n(t) = 0$  and  $f_1 - f_2 = .5/T_b$ . Show that assuming  $s(t) = A_c \cos(2\pi f_1 t)$  and neglecting very small terms,  $Y_1 = .5A_c T_b$  and  $Y_2 = 0$ . Show that for  $s(t) = A_c \cos(2\pi f_2 t)$  and neglecting very small terms we get  $Y_1 = 0$  and  $Y_2 = .5A_c T_b$ . Explain why  $T = 0$  is the appropriate threshold setting in this case.
- (b) Find  $Y_1$  and  $Y_2$  for  $s(t) = A_c \cos(2\pi f_1 t)$  and for  $s(t) = A_c \cos(2\pi f_2 t)$  assuming  $f_1 - f_2 = .25/T_b$  and neglecting very small terms. What should  $T$  be in this case?
- (c) Assume  $f_1 - f_2 = .5/T_b$  and that  $n(t)$  is a white Gaussian noise process with PSD  $.5N_0$ . Then  $N_1$  and  $N_2$  are both Gaussian random variables with mean zero and variance  $.25N_0 T_b$ . Moreover,  $N_1 - N_2$  is a Gaussian random variable with mean zero and variance  $.5N_0 T_b$ . Based on this, find  $p(\text{“1” detected}|\text{“0” sent})$  and  $p(\text{“0” detected}|\text{“1” sent})$  for this FSK system assuming a threshold  $T = 0$ .
- (d) Show that if “1”s and “0”s are equally likely to be transmitted, then  $P_b = Q(\sqrt{E_b/N_0})$ . Evaluate  $P_b$  for  $E_b/N_0 = 10$  dB and for  $E_b/N_0 = 6$  dB.

### Extra Credit Problems

5. (15 points Extra Credit) Assume a BPSK demodulator with an additional additive noise term at the input to its decision device, as shown in the figure below. The decision device outputs a “1” if its input  $r(t)$  has  $\text{Re}[r(t)] \geq 0$ , and a “0” otherwise. Suppose the additive noise term  $n(t) = 1.1e^{j\theta}$ , where  $p(\theta = n\pi/3) = 1/6$  for  $n = 0, 1, 2, 3, 4, 5$ . What is the probability of making a decision error in the decision device, i.e. outputting the wrong demodulated bit, assuming  $A_c = 1$  and that information bits corresponding to a “1” ( $s(t) = \cos(2\pi f_c t)$ ) or a “0” ( $s(t) = \cos(2\pi f_c t + \pi)$ ) are equally likely. Assume  $f_c \gg 1$ .



6. (15 points Extra Credit) Text, Problem 10.29.