

EE 179: Problem Set # 5 - Winter 2007-08

This problem set is last year's midterm. You are *strongly* encouraged to try to do it first on your own with a 90 minute limit. Then it is OK to work cooperatively.

The set is due in class on Wednesday 2/20/08 and late submissions will not receive credit (so we can release the solutions Wednesday afternoon). The Midterm is scheduled for Friday 22 February 11-12:30 in the usual classroom. Open course notes and handouts are permitted (but not the text).

(25 points) 1. Recall the periodic power signal considered in Problem Set 1:

$$x(t) = \begin{cases} -1 & -1 \leq t < 0. \\ +1 & 0 \leq t < 1 \end{cases}$$

and $x(t+2) = x(t)$ for all t .

- (a) (5 points) Find the power spectral density $S_x(f)$ for $x(t)$.
- (b) (10 points) Suppose this periodic wave is passed through a channel with frequency response

$$H(f) = \begin{cases} 2 & 0 \leq |f| \leq 1/4 \\ 1 & 1/4 < |f| \leq 3/4 \end{cases}$$

and the output signal is $y(t)$.

Find the power spectral density $S_y(f)$ of the filter output.

- (c) (5 points) Form the signal $z(t) = y(t) \cos(2\pi 100t)$. Find the spectrum $Z(f)$ and give a labeled sketch.

(40 points) 2. Cheap AM demodulation

This problem considers a variation on the AM modulator considered in class and homework as a means of testing Fourier methods while showing the ideas behind the cheapest classical AM receivers.

Let $x(t)$ be an energy signal with a spectrum $X(f)$ as shown in the figure. The

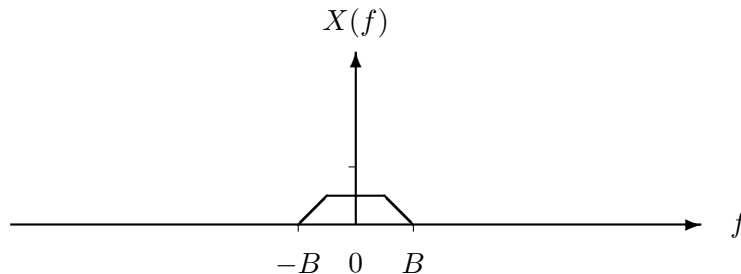


Figure 1: Input signal spectrum $X(f)$

signal is modulated to form a new signal

$$x_c(t) = A(1 + mx(t)) \cos(2\pi f_c t)$$

It is assumed that $A > 0$, $f_c \gg B$, and that m is chosen small enough so that

$$1 + mx(t) \geq 0$$

for all t . Define the squarewave $p(t)$ by

$$p(t) = \begin{cases} +1 & \text{if } \cos(2\pi f_c t) \geq 0 \\ 0 & \text{if } \cos(2\pi f_c t) < 0 \end{cases}$$

Define the signal $y(t)$ by

$$y(t) = x_c(t)p(t).$$

- Find the spectrum $X_c(f)$ in terms of $X(f)$ and sketch the result.
- Find the Fourier series and the Fourier transform of $p(t)$.
- Find the spectrum $Y(f)$ in terms of $X(f)$ and provide a labeled sketch of the result.
- Suppose that $y(t)$ is passed through a DC block and low pass filter $h(t)$ with spectrum

$$H(f) = \begin{cases} 1 & |f| \leq W \\ 0 & \text{otherwise} \end{cases}$$

(A DC block is a linear filter which passes everything except the DC component.) Show that by proper choice of W the output will have the form $cx(t)$, where c is a constant you must find. Give upper and lower bounds on W for this scheme to work.

- Finally, show that multiplication by $p(t)$ is mathematically equivalent to simply rectifying the received signal $x_c(t)$, that is,

$$y(t) = \begin{cases} x_c(t) & \text{if } x_c(t) \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

That is, this demodulator does not really need a multiplier or knowledge of the carrier phase, it needs simply a rectifier, which could be a simple diode.

What happens if the assumption $1 + mx(t) \geq 0$ does not hold?

(35 points) 3. *Mixture and Product Processes*

Let $X(t)$ and $Y(t)$ be WSS processes with $\mu_X = 0, R_X(\tau)$ and $\mu_Y = 4, R_Y(\tau)$, respectively. We define a new process $Z(t)$ for *all* t in the following way:

$$Z(t) = \begin{cases} X(t), & \text{w.p. } 3/4 \\ Y(t), & \text{w.p. } 1/4 \end{cases}$$

In words, nature flips a biased coin at the beginning of time. With probability $3/4$ she sends the random process $X(t)$ forever, with probability $1/4$ she sends $Y(t)$ forever.

- (10 points) Find the mean and autocorrelation functions for $Z(t)$.
- (5 points) Is $Z(t)$ a WSS process? Explain.
- (5 points) If $X(t)$ and $Y(t)$ are both Gaussian, is $Z(t)$ Gaussian? Explain.

Suppose we define another random process $V(t) = X(t)Y(t)$, where we now assume that $X(t)$ and $Y(t)$ are mutually independent (that is, all collections of samples from one process are independent of collections of samples from the other). but have the same means and autocorrelations as listed above.

- Find the mean and autocorrelation of $V(t)$. (10 points)
- Is $V(t)$ a WSS process? If yes, write a formula for the power spectral density. If not, explain why. (5 points)