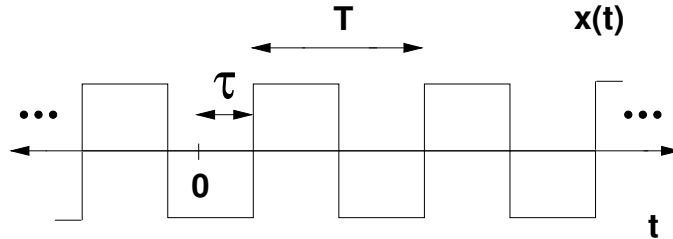


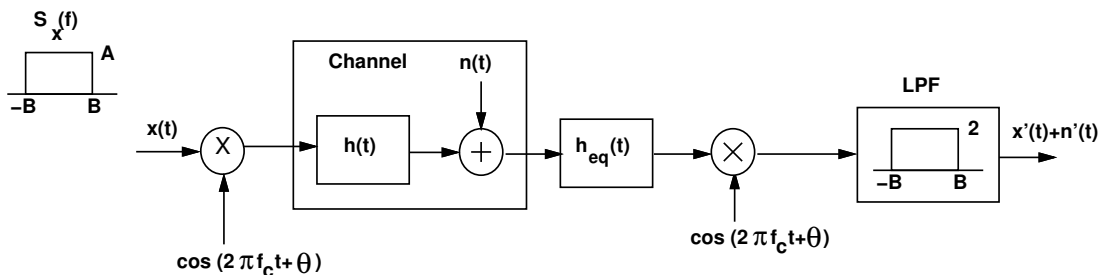
Homework 4: Due Thursday 5/7 at 4pm (no late HWs).

1. (20 points) Let $x(t)$ be the infinite pulse train shown below with period T and random time shift τ , where τ is uniformly distributed between $[0, T]$.



Note that this process is somewhat similar to the process in Example 13 of Chapter 8.

- What is the expectation of $x(t)$?
 - What is the autocorrelation $R(t_1, t_2)$?
 - Show that this is a wide-sense stationary process.
 - Plot $R_x(\tau)$.
 - What is the PSD of this random process?
 - Suppose the process $x(t)$ is passed through an ideal low pass filter with cutoff frequency $1/T$. What is the PSD of the random process output from the filter?
2. (20 points) Consider the communication system shown below. The PSD of $x(t)$ is shown in the figure, and we assume $B \ll f_c$. The noise $n(t)$ is white noise with PSD $S_n(f) = N_0/2$. The cosine has a phase θ that is uniformly distributed on $[0, 2\pi]$ and is independent of $n(t)$. The system output consists of two components: $x'(t)$ corresponding to the input $x(t)$ alone and $n'(t)$ corresponding to the noise $n(t)$ alone.



Suppose the channel has frequency response

$$H(f) = \begin{cases} 1/3 & |f - f_c| \leq B/2 \\ 1 & B/2 < |f - f_c| < B \\ 0 & \text{else} \end{cases}$$

- (a) Find the power in $x(t)$.
- (b) Find the equalizer frequency response $H_{eq}(f)$ for $|f - f_c| < B$ so that $x'(t) = x(t)$.
- (c) Sketch the PSD of $n'(t)$ and find its power for $H_{eq}(f)$ from part (b).
- (d) Find the SNR at the lowpass filter output (ratio of signal power over noise power). Compare with the SNR assuming $h(t) = h_{eq}(t) = \delta(t)$ (i.e. no noise enhancement).
3. (20 points) Let $X(t)$ be a WSS white Gaussian random process with mean zero and PSD $S_X(f) = N_0/2$. Let $X = \int_0^T X(t)dt + .5 \int_0^{2T} X(t)dt$. Let $Y(t) = X(t) * h(t)$ for $h(t) = \text{sinc}(t/T)$.
- (a) Find the mean and variance of X and also the probability $P(X \leq 3)$ in terms of the error function.
- (b) Find the PSD and autocorrelation of $Y(t)$. Is $Y(t)$ white noise?
- (c) What is the minimum time separation τ such that samples of $Y(t)$ are independent?
- (d) What is $E[Y(3)Y(4.5)]$ for $T = N_0 = 1$?
4. (20 points) Text, Problem 8.36
5. (20 points) Let $X(t)$ and $Y(t)$ be independent wide sense stationary Gaussian random processes with $E[X(t)] = E[Y(t)] = 0$, $R_X(\tau) = 10\delta(\tau)$, and $R_Y(\tau) = 5\delta(\tau)$. What is the distribution of

$$Z = \int_0^2 [X(t) - 3Y(t) + 10]dt.$$