

Homework 3: Due Thursday 4/29 at 4pm.

1. (15 points) Text, Chapter 8, Problem 8.19 and 8.20.
2. (15 points) Text, Chapter 8, Problem 8.22.
3. (15 points) Text, Chapter 8, Problem 8.24.
4. (20 points) an experiment with sample space the unit interval $(0, 1)$ and a probability measure described by a uniform probability density function (pdf) $f(u) = 1$ for $0 \leq u \leq 1$ and zero otherwise.

In each part of the problem a random variable is defined on this probability space Find the cumulative distribution functions (cdfs) and the pdf's for the following random variables:

- (a) $X = X(u) = |u|^2$. The cdf is defined by $F_X(x) = \Pr(X \leq x)$ and the pdf is defined by $f_X(x) = dF_X(x)/dx$.
- (b) $Y = Y(u) = u^{1/2}$,
- (c) $Z = Z(u) = \ln u$,
- (d) $V = aZ + b$, where a and b are fixed constants.
- (e) Find the probability mass function $p_W(w) = \Pr(W = w)$ and the cdf for the random variable $W = W(u)$ defined by

$$W(u) = \begin{cases} -1 & 0 \leq u \leq 0.5 \\ +1 & .5 < u \leq 1 \end{cases}$$

5. (15 points) Text, Chapter 8, Problem 8.25.
6. (20 points) Let X_1 and X_2 be two independent random variables, where X_1 is a Gaussian random variable with mean μ_1 and variance σ_1^2 , and X_2 is a Gaussian random variable with mean μ_2 and variance σ_2^2 .
 - (a) Find the characteristic functions for X_1 and X_2 .
 - (b) Find the characteristic function for their sum $Y = X_1 + X_2$.
 - (c) Find the distribution for $Y = X_1 + X_2$. What is this distribution?
 - (d) Can you generalize this result to a sum of N independents Gaussian random variables, with means μ_1, \dots, μ_N and variance $\sigma_1^2, \dots, \sigma_N^2$?