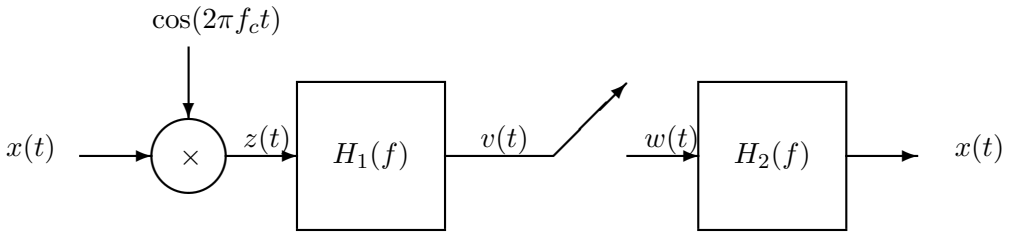
EE179 Winter 2007–2008

Introduction to Communications

Problem Set #2

Due: Thursday, January 31 at 5 PM.

1. (20 points) *Demodulation by Sampling.* This problem illustrates a creative form of demodulation using a sampler. Consider the communication system shown below, where the signal $w(t)$ is obtained by sampling the channel output $v(t)$ every T seconds.



Suppose the information signal $x(t)$ has Fourier transform

$$X(f) = \begin{cases} .5 & |f| < B/2 \\ 1 & B/2 \le |f| < B \\ 0 & else \end{cases},$$

the channel has frequency response

$$H_1(f) = H_1(-f) = \begin{cases} 2 & |f - f_c| < B/2 \\ 1 & B/2 \le |f - f_c| < B \\ 0 & else \end{cases},$$

and the carrier frequency f_c is much bigger than the bandwidth B of $X(f)$.

(a) Sketch $Z(f)$ and $V(f)$.

(b) Sketch $W(f)$ and $w(t)$ for $T = 1/f_c$.

(c) Find $H_2(f)$ so that $y(t) = x(t)$ for $T = 1/f_c$.

(d) For $H_2(f)$ from part (c), for what values of T other than $1/f_c$, if any, will $y(t) = cx(t)$ for some constant c?

2. (20 points)

   (a) Let $z(t) = x(t)\cos^2(2\pi f_c t)$ where $x(t)$ is a power signal with $S_x(f) = A_c^2$ for $|f| \leq B$ and $S_x(f) = 0$ for $|f| > B$. Assume $B << f_c$. Sketch $S_z(f)$ and find its power.

   (b) For the $z(t)$ defined in part (a), let $w(t) = z(t)\text{sinc}(Bt)$. Sketch $S_w(f)$, find its power, and find the smallest τ such that $R_w(\tau) = 0$.

3. (20 points) Given a signal $x(t) = \text{sinc}(t)$, find and sketch its energy spectral density $\phi_x(f)$ and its autocorrelation $R_f(\tau)$.

4. (20 points) As an example of an application of autocorrelation. Suppose that you receive a known energy signal $x(t)$, but with an unknown delay t_0 which you wish to estimate (perhaps it will indicate the distance to a target). Suppose you have a variable delay device so you can produce $x(t-\tau)$. By multiplying your actual received signal $x(t-t_0)$ by your local signal $x(t-\tau)$ with a variable delay and integrating the result, show how you can estimate t_0.

   Now suppose that $x(t)$ is a periodic signal with unknown period T_0. How might you estimate the period based on the autocorrelation? (something like this is done for pitch estimation in speech processing).

5. (10 points) Book, Problem 2.39.

6. (10 points) Book, Problem 2.45.