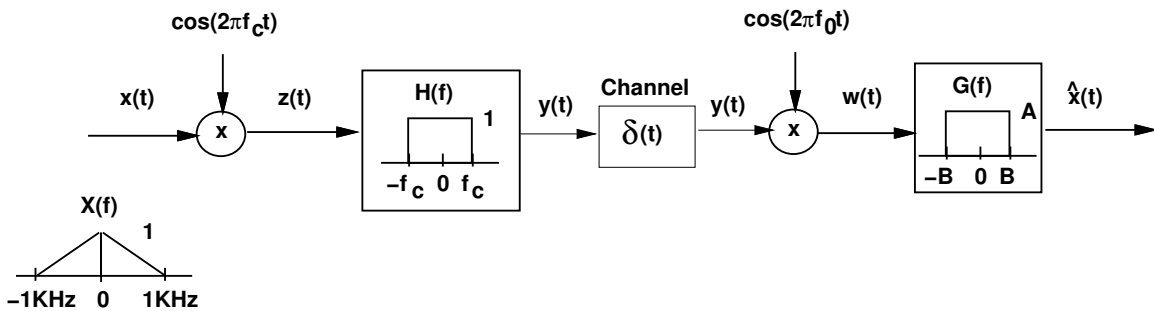


Homework 2: Due Thursday 4/23 at 4pm.

1. (15 points) Bandwidth Efficient Modulation.

This problem illustrates a form of modulation that uses less spectrum than standard modulation by exploiting the symmetry of real signals. Consider the communication system shown in the figure below. The signal $x(t)$, with spectrum $X(f)$ as shown in the figure, is multiplied by a cosine of frequency $f_c = 10$ KHz to obtain the signal $z(t)$. Then $z(t)$ is passed through a low-pass filter $H(f)$ with cutoff frequency f_c ($H(f) = 1$ for $|f| \leq f_c$ and zero otherwise) to produce the transmitted signal $y(t)$. Assuming the channel is a wire, the received signal is also $y(t)$, which in the receiver is multiplied by a cosine of frequency f_0 . The resulting signal $w(t)$ is passed through a low-pass filter $G(f)$ with cutoff frequency B and gain A ($G(f) = A$ for $|f| \leq B$ and zero otherwise). The signal at the output of the filter $G(f)$ is denoted by $\hat{x}(t)$.



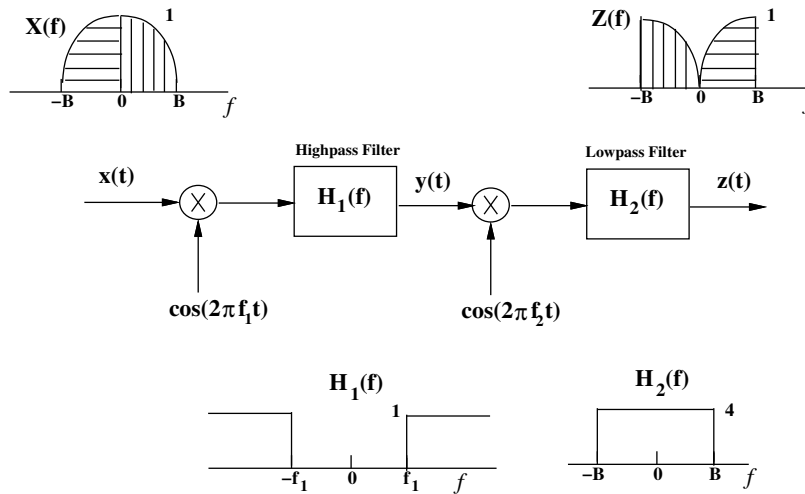
- (a) Sketch $Z(f)$.
- (b) Sketch $Y(f)$ (this is the modulated signal).
- (c) Find the values of f_0 , B , and A such that $x(t) = \hat{x}(t)$ (so we recover the original signal from the modulated signal). Also, sketch the spectrum $W(f)$ of the signal $w(t)$ that results from this choice of f_0 .
- (d) Repeat part (c) assuming we remove the filter $H(f)$ (so $z(t) = y(t)$). Compare the passband bandwidth of the transmitted signal $y(t)$ without $H(f)$ versus the passband bandwidth of $y(t)$ with $H(f)$ (i.e. the bandwidth of $Y(f)$ found in part (b)).

2. (15 points) Voice Scrambling.

One of the big advantages of digital modulation is that signals can be easily encrypted using standard encryption techniques. Most analog communication signals can be easily listened to using an antenna and simple AM receiver. This problem explores a simple encryption strategy for analog voice signals that distorts the transmitted signal to make eavesdropping more difficult.

Consider the voice scrambling system shown below, with input voice signal $X(f)$ as shown in the figure. The highpass filter $H_1(f)$ passes all frequency components above f_1 and removes all frequency components below f_1 . The low pass filter passes all frequency components in $[-B, B]$ and removes all other frequency components.

- (a) Sketch $Y(f)$ assuming $f_1 \gg B$.



(b) Find f_2 as a function of f_1 and B such that $Z(f)$ is as shown in the figure.

(c) Explain how you would descramble the scrambled signal, i.e. how you would recover $X(f)$ from $Z(f)$.

3. (15 points) Text, Chapter 2, Number 2.38

4. (10 points) Text, Chapter 2, Number 2.41

5. (15 points) Text, Chapter 2, Number 2.47

6. (15 points) Text, Chapter 2, Number 2.49

7. (15 points) We have seen in class that modulating a narrowband power signal shifts its PSD by the carrier frequency. We will show in this problem that when we modulate twice, we don't necessarily just shift the signal twice, since the signal resulting from multiplying the narrowband signal with the first cosine is not necessarily narrowband relative to the second cosine. Let $x(t)$ be a power signal with power spectral density $S_x(f)$ that is bandlimited to $[-B, B]$. Let $y(t) = x(t) \cos(2\pi f_1 t) \cos(2\pi f_2 t)$.

(a) Find the power spectral density of $y(t)$.

(b) Find conditions on f_1 and f_2 such that the power spectral density of $y(t)$ is equal to

$$S_y(f) = A[S_x(f - \Delta f) + S_x(f + \Delta f) + S_x(f - f_1 - f_2) + S_x(f + f_1 + f_2)],$$

where $\Delta f = f_1 - f_2$ and A is some constant.

(c) Find A .

(d) What is $S_y(f)$ when $\Delta f = 0$?