EE 179 - Introduction to Communications - Spring 2009

Homework 1: Due Thursday 4/16 at 4pm .

1. LTI Filtering of Periodic Signals (10 points): Consider an LTI filter with frequency response $H(f) = \sin(8\pi f)/(2\pi f)$. If the input to this filter is a periodic signal with fundamental period $T_0 = 8$ that is defined over one period as

$$g_{T_0}(t) = \begin{cases} 1 & 0 \leq t < 4 \\ -1 & 4 \leq t < 8 \end{cases},$$

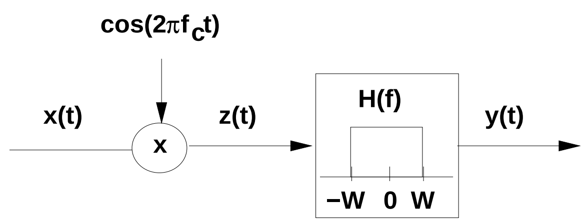
   determine the corresponding filter output $y(t)$.

2. Parseval's Relation(10 points): Show that $\int_{-\infty}^{\infty} sinc^2(t) = 1$

3. Squared Sinusoids (10 points): Find the Fourier transform of the squared sinusoidal functions $\cos^2(2\pi f_c t)$ and $\sin^2(2\pi f_c t)$

4. Fourier Transform Properties (15 points): Text, Problem 2.19 (page 91).

5. Modulation (20 points): Consider the system shown below. The input $x(t) = \text{sinc}^2(1000t)$ is multiplied by a cosine at center frequency $f_c = .5$ KHz and then sent through a low pass filter $H(f)$ which passes all frequencies below $W = .5$ KHz.



   (a) Find $X(f)$.

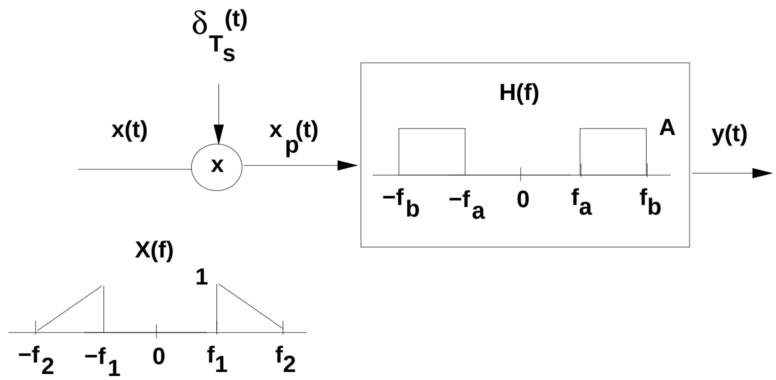
   (b) Sketch $Z(f)$, the Fourier transform of $z(t) = x(t)\cos(2\pi f_c t)$.

   (c) Sketch $Y(f)$, the Fourier transform of the filter output.

   (d) Find $y(t)$.

6. Bandpass Sampling (15 points): Consider the bandpass signal $x(t)$ shown below where $X(f) = 0$ except for $f_1 \leq |f| \leq f_2$. Assume that $f_1 > f_2 - f_1$. According to the sampling theorem we would need to sample $x(t)$ at $T_s \leq .5/f_2$ to recreate $x(t)$ from its samples. However it appears that we should be able to sample at a lower rate since the spectrum is nonzero only in the frequency range between f_1 and f_2. This problem shows that indeed we can sample at a lower rate that depends only on the bandwidth occupied by the signal. In particular, let $f_2 = 10$ MHz and $f_1 = 5$ MHz. Suppose we sample $x(t)$ at a rate $T_s = .5/(f_2 - f_1)$ to obtain the sampled signal $x_s(t)$. We then pass $x_s(t)$ through the bandpass filter $H(f)$ shown in the figure that has a value of $.5A$ at the bandedges (i.e. exactly at frequencies $\pm f_b$ and $\pm f_a$).

   (a) Sketch $X_s(f)$.



(b) Determine the constants A, f_a, and f_b such that the output of the filter $H(f)$ equals the original signal $x(t)$.

7. Frequency Sampling (20 points): Let

$$X(f) = \sum_{n=-2}^{2} \delta(f - nf_0)$$

for $f_0 = 10$ KHz.

(a) Sketch $X(f)$.

(b) Sketch $x(t) = \mathcal{F}^{-1}[X(f)]$.

(c) Find the exponential Fourier Series coefficients of $x(t)$.