EE179 Spring 2008–2009

Introduction to Communications

Problem Set #6 Solutions

1. (15 points) Text, Chapter 3, Number 3.26.
   Answer
   The signal $m(t) = A\mathrm{rect}(\frac{t}{T})$. After passing the signal through SSB the modulator, the modulated signal $x(t) = A_c m(t)\cos(2\pi f_c t) - A_c \hat{m}(t)\sin(2\pi f_c t)$, where $\hat{m}(t)$ is the hilbert transform of $m(t)$ and $\hat{m}(t) = m(t) * \frac{1}{\pi t}$.

   Since the envolope of the singal is $\sqrt{m^(t) + \hat{m}^2(t)}$, and $m(t)$ is constant from the beginning of the pulse till the end of the pulse, the peak occurs when $\hat{m}(t)$ is at its peak or bottom. Since $\hat{m}(t) = m(t) * \frac{1}{\pi t}$, we can see from the convolution that $|\hat{m}(t)|$ is at its maximum at the beginning and end of the pulse, which proves that the envolpe exhibits peaks at the beginning and end of the pulse.

2. (20 points) *Answer*

   (a)
   $$P_m = \int_{-\infty}^{\infty} S_m(f)df = 15B\mathrm{mW/HZ}.$$

   (b) The PSD of $S_{USB}(f)$ contains the upper sidebands of Sm(f), scaled by $A_c^2/4$. Since $H(f)H_{eq}(f) = 1$, $X(f) = S_{USB}(f)$. Upon multiplying by the second cosine term, since only the upper sidebands are present, they do not overlap when shifted up and down by f_c. Therefore, the PSD of $Y(f)$ is simply $S_x(f)$ shifted up and down by f_c, then scaled by 1/4. The LPF eliminates the terms at $\pm f_c$, so $S_{m'}(f) = A_c^2 S_m(f)/16$, and

   $$P_{m'} = \frac{A_c^2}{16}(15B\mathrm{mW}) = \frac{15A_c^2B}{16}\mathrm{mW}$$

   The PSD of $n(t)$ after demodulation is $.25(N_0/2) + .25(N_0/2) = N_0/4$, so the output noise power after filtering is

   $$P_{n'} = 0.25N_0B = 0.1B\mathrm{mW}.$$

   So we find that the output SNR is

   $$\frac{15A_c^2B/16}{0.1B} = \frac{75A_c^2}{8}.$$

(c) $H_{eq}(f)$ must be chosen so that $H(f)H_{eq}(f) = 4/A_c$ (this scaling factor is required to offset the factor of .5Ac introduced by the SSB modulator, and the factor of .5 introduced when multiplying by the second cosine).

$$H_{eq}(f) = \begin{cases} 40/A_c & f_c - 0.5B \leq |f| \leq f_c + 0.5B \\ 4/A_c & otherwise \end{cases}.$$

Since $m_0(t) = m(t)$, $P_{m_0}(f) = P_m(f) = 15B$ mW. The noise PSD after equalization is $.5N_0|Heq(f)|^2$. The output noise PSD is given by

$$.25[.5N_0|H_{eq}(f - fc)|^2 + .5N_0|H_{eq}(f + fc)|^2] = .25N_0|H_{eq}(f - fc)|^2, |f| < B.$$

So the output noise power is:

$$P_{n_0} = .25N_0(1600B/A_c^2 + 16B/A_c^2) = \frac{404B}{5A_c^2}\text{mW}.$$

And the output SNR is

$$\frac{P_{m'}}{P_{n'}} = \frac{75A_c^2}{404}.$$

With $H_{eq}(f) = 1$, the output noise power is .1B mW as obtained in part (b), since $H(f)$ does not affect the noise. Now looking at the message power, $S_{m_0}(f) = A_c^2 16 S_m(f)|H(f - f_c)|2$, therefore

$$P_{m_0} = \frac{A_c^2}{16}[(10mW)(.12)B + (5mW)(12)B] = \frac{51A_c^2 B}{160}\text{mW}$$

And the output SNR is now

$$\frac{P_{m'}}{P_{n'}} = \frac{51A_c^2}{16}.$$

(d) The design of the equalizer entails a tradeoff between offsetting the effects of channel distortion and maximizing the output SNR. As shown in part (c), a perfect equalizer eliminates all distortion introduced by the channel, but at the expense of a low SNR. Without an equalizer, the SNR is approximately 17 times as high, but the message is distorted at the output.

3. (15 points) Text, Chapter 3, Number 3.27 *Answer*

(a) The message signal after passing a USSB modulator is shown in Fig. 1. Demodulating it with a local oscilator of 100.2 kHz is just to shift the spectrum of the modulated signal up and down by 100.2 kHz and passing it through a low pass filter. The demodulated signal is shown in Fig. 2

(b) Following similar steps as in part (a), the demodulated signal is shown in Fig. 3, which has frequency components at $\pm 120, \pm 220, \pm 420$ Hz.

4. (15 points) Text, Chapter 3, Number 3.32 *Answer*

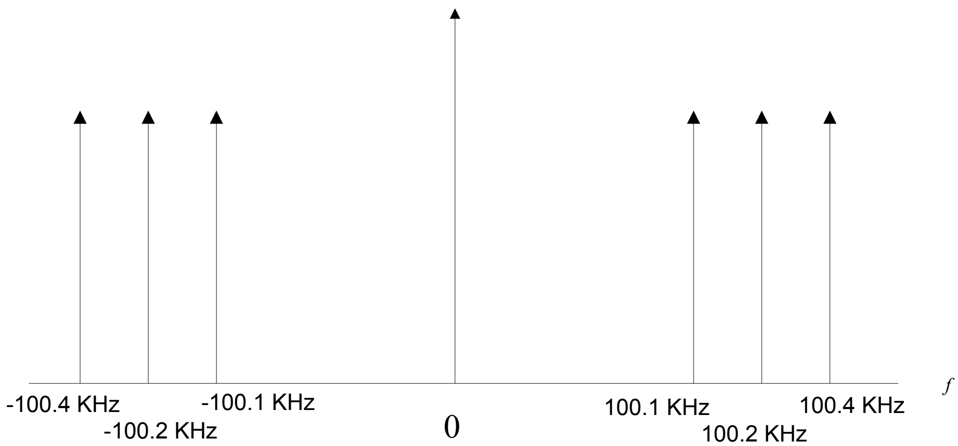

Figure 1: USSB modulated signal

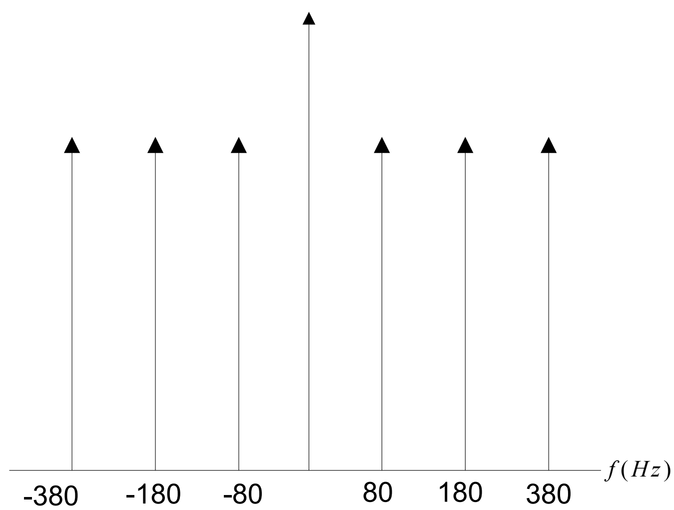

Figure 2: demodulated signal (USSB)

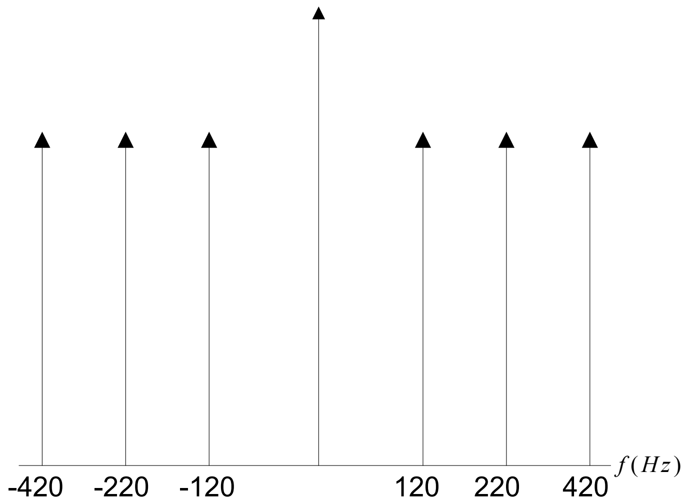

Figure 3: demodulated signal (LSSB)

(a) Write h(t) in the Quadature carrier form $h(t) = h_I(t)\cos(2\pi f_c t) - h_Q(t)\sin(2\pi f_c t)$ and take Fourier transform on both sides, we have:

$$H(f) = \frac{H_I(f-f_c) + H_I(f+f_c)}{2} - \frac{H_Q(f-f_c) - H_Q(f+f_c)}{2j}.$$

Apply equation (3.26) in the text book for $-W \leq f \leq W$ and note that the two $H_Q(f)$'s in $H(f)$ cancels each other out at baseband, we obtan $H(f-f_c) + H(f+f_c) = H_I(f)$ for $-W \leq f \leq W$. Therefore $H_I(f) = 1$, for $-W \leq f \leq W$.

(b) Since $h(t) = h(t)^*$, we have $H(f) = -H^*(-f)$. Therefore $H_Q(-f) = -H_Q(f)$ for $-W \leq f \leq W$. Let $f = 0$, $H_Q(-0) = -H_Q(0)$, we have $H_Q(0) = 0$. For $f_v + f_c \leq f \leq f_c + W$, $H(f) = \frac{H_I(f-f_c)}{2} - \frac{H_Q(f-f_c)}{2j} = 1$. Therefore $H_Q(f-f_c) = 1$ for $f_v + f_c \leq f \leq f_c + W$, i.e., $H_Q(f) = 1$ for $f_v \leq f \leq W$.

5. (15 points) Text, Chapter 4, Number 4.13. *Answer*

(a) Given the frequency sensitivity, we can calculate the maximum frequency deviation as $\Delta f = k_f A_m = (25kHz/V)(20V) = 500kHz$. Using Carsons rule gives us $B = 2\Delta f(1 + \frac{1}{\beta}) = 2\Delta f(1 + \frac{f_m}{\Delta f}) = 1.2MHz$.

(b) $\beta = 5$ from part (a). Consulting Fig. 4.9, we see that the corresponding value for $B/\Delta f$ is approximately 3.5. The resulting value of B is thus (3.5)(500kHz) = 1.75MHz.

(c) $\beta = 10$ after the change. Carsons rule yields a bandwidth of 2.2 MHz, while the 99 percent bandwidth method yields a bandwidth of 3 MHz. This problem illustrates a characteristic of FM not found in AM. Namely, the bandwidth of the modulated waveform is dependent on the amplitude of the message signal, which is not true of AM.

(d) $\beta = 2.5$ after the change. Carsons rule gives a transmission bandwidth of 1.4 MHz, and the 99 percent bandwidth method yields a transmission bandwidth of 2 MHz. We can

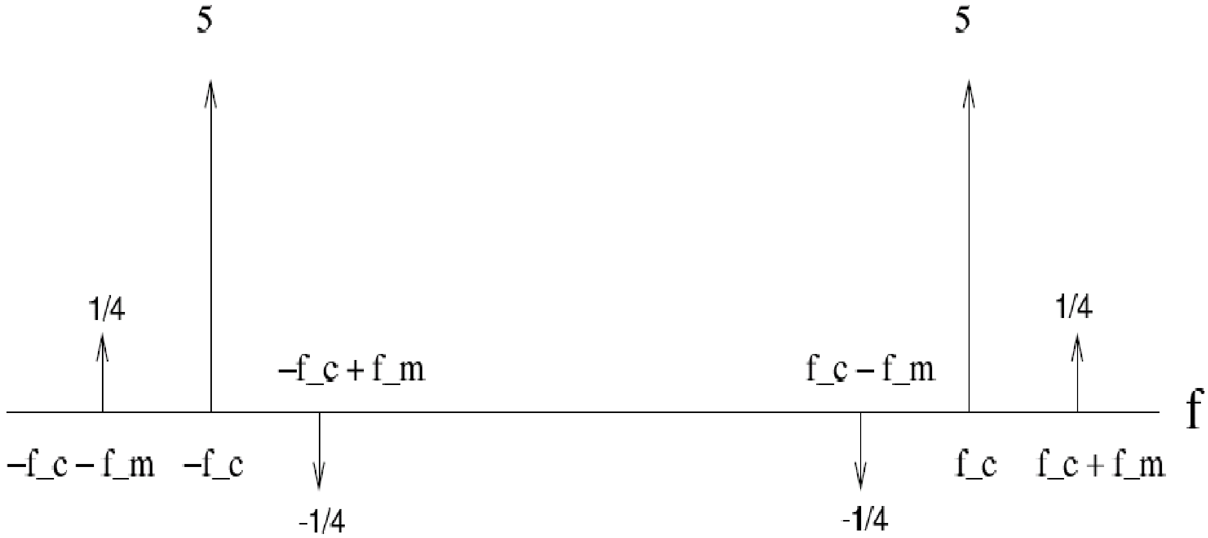

Figure 4: $S(f)$ for $k_f = 10$

draw a few observations here. Carsons rule generally underestimates FM transmission bandwidth, while the 99 percent bandwidth method yields a conservative estimate. So the practical bandwidths for the waves described above lie somewhere between the two figures. Lastly, we see that the transmission bandwidth of an FM wave is much more sensitive to the amplitude of the message signal than it is to the bandwidth of the message. Doubling the amplitude of the message signal in part (c) yielded a much larger increase in transmission bandwidth than doubling the message signal bandwidth in part (d).

6. (20 points) *Answer*

(a) We calculate that $\beta = \Delta f/f_m = k_f A_c/f_m = 0.1$. Since $\beta < 0.3$, we can make the approximation that $J_0(0.1) = 1$ and $J_1(0.1) = 0.5(0.1) = 0.05 = -J - 1(0.1)$ and all other Bessel coefficients are zero. We therefore have

$$s(t) = A_c[J_0(0.1)\cos(2\pi f_c t) + J_1(0.1)\cos(2\pi(f_c + f_m)t) + J_{-1}(0.1)\cos(2\pi(f_c - f_m)t)],$$

i.e.,

$$s(t) = 10[\cos(2\pi f_c t) + 0.05\cos(2\pi(f_c + f_m)t) - 0.05\cos(2\pi(f_c - f_m)t)]$$

The spectrum of s(t) is sketched in Fig. 4. The transmission bandwidth of the FM wave is estimated as $2\Delta f(1 + \frac{1}{\beta})) = 2(100)(1 + 10) = 2.2$kHz while a direct inspection of the spectrum would yield a bandwidth of $2f_m = 2$kHz.

(b) Now $k_f = 250$ and $\beta = 2.5$. From the graph given, we can read off the values for the Bessel coefficients. $J_0(2.5) \approx -0.1$, $J_1(2.5) = -J_{-1}(2.5) \approx 0.5$, $J_2(2.5) = J_{-2}(2.5) \approx 0.45$, $J_3(2.5) = -J_{-3}(2.5) \approx 0.2$, $J_4(2.5) = J_{-4}(2.5) \approx 0.5$. All other Bessel coefficients are approximated as zero. The spectrum of s(t) is sketched in Fig. 5. Carsons Rule

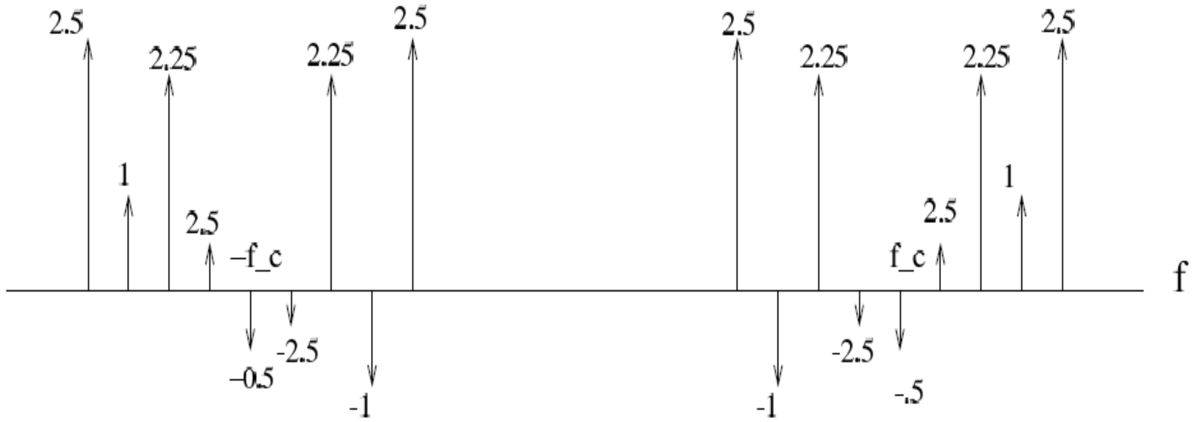

Figure 5: $S(f)$ for $k_f = 250$

gives an approximate bandwidth of 7 kHz, while direction examination of the spectrum gives a bandwidth of $8f_m$, or 8 kHz.