EE 179: Introduction to Communications - Winter 2006

Final Exam

This exam is open book and notes, and any results derived in class or in the book can be used without proof. A plot of the Q function is given at the end of the exam. Your plot sketches should indicate the general shape of the function and all key values on the x and y axes, including zero crossings. Graded exams and solutions will be available next week. Good luck, and have a great break!

Problem 1 (40 points): Short Answer.

Provide a short answer for each question below

(a) If a BPSK system has a bit error probability of .001 for $R = 10$ Mbps, and the data rate is doubled to $R = 20$ Mbps with the transmit and noise power remaining the same, what is the new probability of bit error?

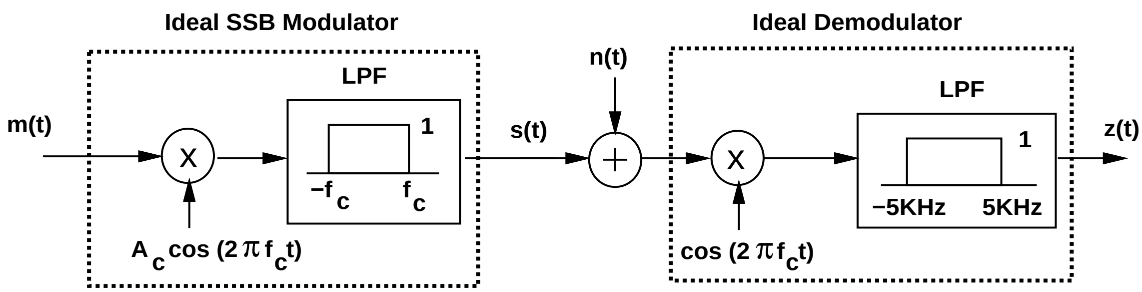
(b) Consider a standard AM signal where for some values of t, $k_a m(t) = -1 - \delta$ for some small positive number δ. Describe the distortion introduced by an envelope detector for this signal over these t values as compared to the distortion introduced by a product demodulator. Neglecting noise, which would sound better in an AM radio receiver and why?

(c) Find the energy in the signal $x(t) = sinc^2(t)$.

(d) Let $n(t)$ be a WSS process with mean zero and PSD $N_0/2$. Given a constant $T \neq 0$, is $n(t) + n(t + T)$ WSS (prove or disprove)?

(e) For $n(t)$ a zero-mean white Gaussian noise with PSD $N_0/2$, find the autocorrelation of $z(t) = n(t) * rect(t)$, where $*$ denotes convolution. Also give **all** times T for which the samples $z(0)$ and $z(T)$ are independent.

Problem 2 (40 points): AM Modulation, Transmit Power Constraints, and SNR. Consider the SSB AM system shown below. We assume an ideal SSB modulator that filters out the upper sidebands to transmit the lower sideband modulated signal $s(t)$. Standard demodulation is used. In general the transmit power, i.e. the power in the transmitted signal $s(t)$, is constrained either by FCC rules or by a certain desired battery life. Assume the information signal $m(t)$ has autocorrelation function $R_m(\tau) = 400 sinc(10000\tau)$.



1. Find A_c such that the power in $s(t)$ is equal to 10 mW.

2. Given A_c found in part (a), find the power in the demodulator output $z(t)$ in the absence of noise (i.e. for $n(t) = 0$).

3. Now assume that $n(t)$ is an AWGN process with power spectral density $S_n(f)$ (in W/Hz) that is symmetric for positive and negative frequencies, and for f positive the PSD is given by $S_n(f) = N_0, |f - f_c| < 2500$, $S_n(f) = .5N_0, 2500 \leq |f - f_c| \leq 5000$, and $S_n(f) = 0, |f - f_c| > 5000$. Find the PSD and power in the demodulator output $z(t)$ due to noise only (i.e. the PSD and power of $z(t)$ for $m(t) = 0$).

4. Given your answers to (b) and (c), find the value of N_0 such that the SNR at the demodulator output equals 10.

5. Repeat parts (a) and (b) assuming you use DSBSC modulation and demodulation ($s(t) = m(t)A_c \cos(2\pi f_c t)$) instead of the SSB modulation and demodulation shown in the figure, with the same power constraint of 10 mW on $s(t)$. What is the SNR at the demodulator output is this case (use the N_0 obtained in part (d), since $n(t)$ is the same as under SSB, so the noise power doesn't change)?

You should find that DSBSC has a higher SNR than SSB given the same transmit power constraint. Thus, although SSB is more spectrally-efficient than DSBSC, you pay a price in SNR when the transmit power is constrained.

Problem 3 (40 points): Detector Design for FM Modulation

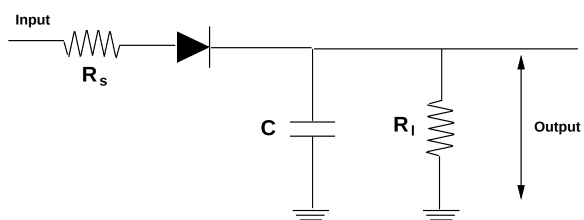
This problem illustrates design choices and limitations for certain FM detector designs. Consider an FM system where the modulated signal is $s(t) = 10\cos\left[2\pi f_c t + 30\pi\int_0^t m(\tau)d\tau\right]$ for a carrier frequency of 100 MHz. Assume the modulating signal $m(t) = 5\cos(2\pi f_m t)$ for $f_m = 25$ KHz.

(a) What is the approximate bandwidth of $s(t)$? Is this NBFM or WBFM?

(b) Find the spectrum of $s(t)$.

(c) Suppose you demodulate $s(t)$ using a discriminator followed by an envelope detector. Over what bandwidth must the discriminator approximate an ideal differentiator? Sketch the frequency response (amplitude and phase) of the discriminator over this bandwidth.

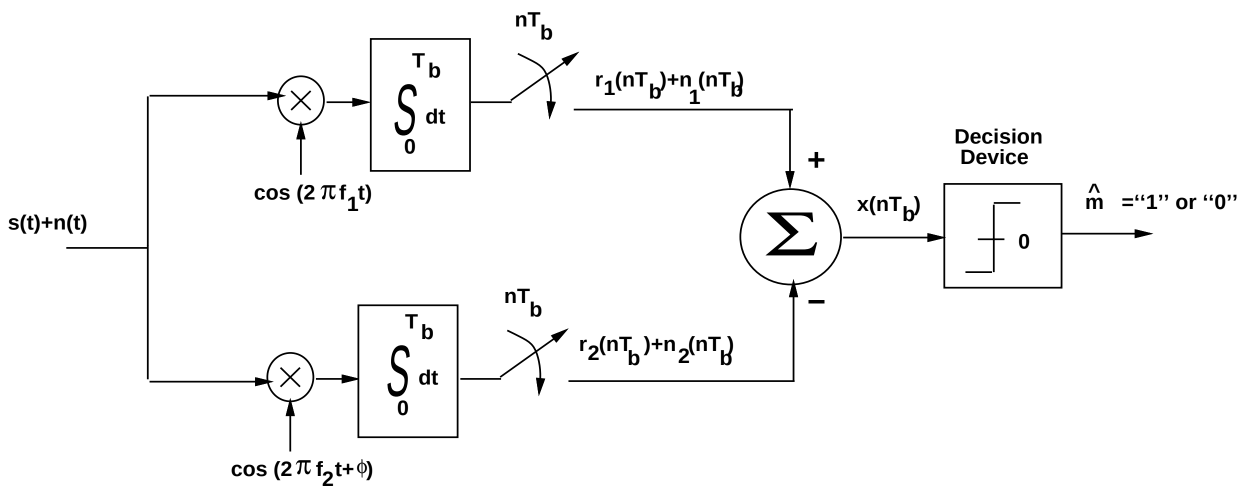
(d) Suppose you demodulate $s(t)$ using an ideal differentiator followed by an envelope detector. Assume a standard envelope detector as shown in the figure below, where the capacitor has capacitance $C = 10^{-9}$F. Propose values for the source resistance R_s and load resistance R_l such that the output of the envelope detector is approximately equal to $c_1 + c_2 m(t)$ for some constants c_1 and c_2.



(e) Suppose you decide to use a zero-crossing detector for $s(t)$. Find the minimum interval T for a zero-crossing detector such that there are at least four zero crossings in every interval T.

Problem 4 (40 points): FSK with Phase Offset

In this problem we consider the impact of a phase offset on the bit error probability of FSK modulation. Consider the FSK system shown below



We have $s(t) = A_c \cos(2\pi f_1 t)$ to send a "1" bit and $s(t) = A_c \cos(2\pi f_2 t)$ to send a "0" bit, with $f_1 - f_2 = .5/T_b$ and $f_1, f_2 >> 1$. This signal passes through a channel with AWGN $n(t)$ of mean zero and PSD $N_0/2$. The received signal is demodulated using standard FSK demodulation, but the carrier associated with the second branch has a phase offset of ϕ. The upper branch generates $r_1(nT_b) + n_1(nT_b)$ after downconversion, integration, and sampling, and the lower branch generates $r_2(nT_b) + n_2(nT_b)$ after these operations. If $r_1(nT_b) + n_1(nT_b)$ is greater than $r_2(nT_b) + n_2(nT_b)$ then the demodulator outputs a "1" bit, otherwise it outputs a "0" bit.

(a) Show that for $\phi = 0$, $n_1(nT_b)$ and $n_2(nT_b)$ are independent.

(b) For $\phi = \pi/8$, find the distribution of $n_1(nT_b)$ and of $n_2(nT_b)$.

(c) Show that for $\phi = \pi/8$, $n_1(nT_b)$ and $n_2(nT_b)$ are not independent.

(d) For $\phi = \pi/8$, find $r_1(nT_b)$ and $r_2(nT_b)$ assuming a "1" bit was sent.

(e) Given that a "1" is sent, for what values of $n_1(nT_b) - n_2(nT_b)$ will a "0" be demodulated? What would the values that cause an error be for $\phi = 0$?

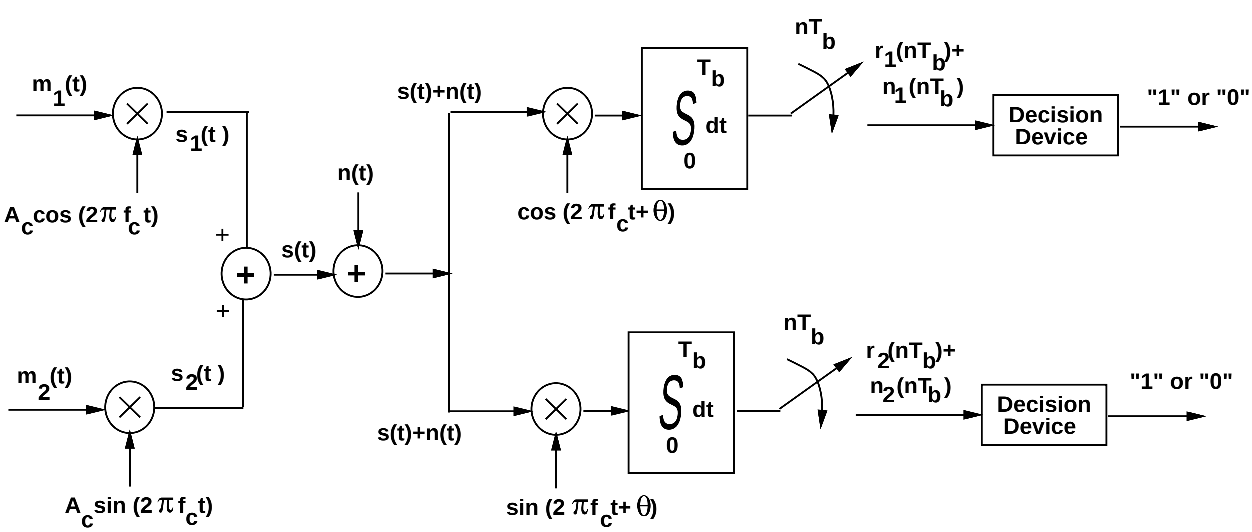
Problem 5 (40 points): Multilevel Modulation with a Phase Offset

ASK, PSK, and FSK are all digital modulation techniques that send one bit at a time. We can double the data rate of digital modulation by sending two different bit streams simultaneously using both sine and cosine carriers, as we illustrate in this problem. Specifically, we examine a modulation which sends two BPSK modulated signals simultaneously, and we examine the degradation in this modulation when the receiver has a carrier phase offset relative to the transmitter.

Consider the digital communication system shown in the figure below, where $m_1(t)$ and $m_2(t)$ are different information signals corresponding to different bit streams ($m_i(t) = 1, nT_b - .5T_b < t \leq nT_b + .5T_b$ if the nth bit is a "1" and $m_i(t) = -1, nT_b - .5T_b < t \leq nT_b + .5T_b$ if the nth bit is a "0"). So $s_1(t)$ and $s_2(t)$ are BPSK modulated waveforms, where the carrier for $s_1(t)$ is $\cos(2\pi f_c t)$ and the carrier for $s_2(t)$ is $\sin(2\pi f_c t)$. The transmitted signal is $s(t) = s_1(t) + s_2(t)$.

The channel introduces AWGN $n(t)$ with mean zero and PSD $N_0/2$, so the received signal is $s(t)+n(t)$. In the receiver the signal is split into an upper and a lower branch. The upper branch is multiplied by the cosine carrier with phase offset θ, integrated, and sampled to get $r_1(nT_b)+n_1(nT_b)$ The decision device in this branch has a threshold of zero such that for $r_1(nT_b) + n_1(nT_b) > 0$, the bit corresponding to $m_1(nT_b)$ is decoded as a "1", and for $r_1(nT_b) + n_1(nT_b) \leq 0$ this bit is decoded as a "0". A similar procedure follows in the lower branch: the signal is multiplied by the sine carrier with phase offset θ, integrated over a bit time, sampled, and passed through a decision device. This device decodes the bit corresponding to $m_2(nT_b)$ as a "1" if $r_2(nT_b) > 0$ and as a "0" if $r_2(nT_b) + n_2(nT_b) \leq 0$.

Assume throughout the problem that $f_c >> 1$. Also, the trig identities $\cos a \cos b = .5[\cos(a+b) + \cos(a-b)]$, $\cos a \sin b = .5[\sin(a+b) - \sin(a-b)]$, and $\sin a \sin b = .5[\cos(a-b) - \cos(a+b)]$ may be useful.

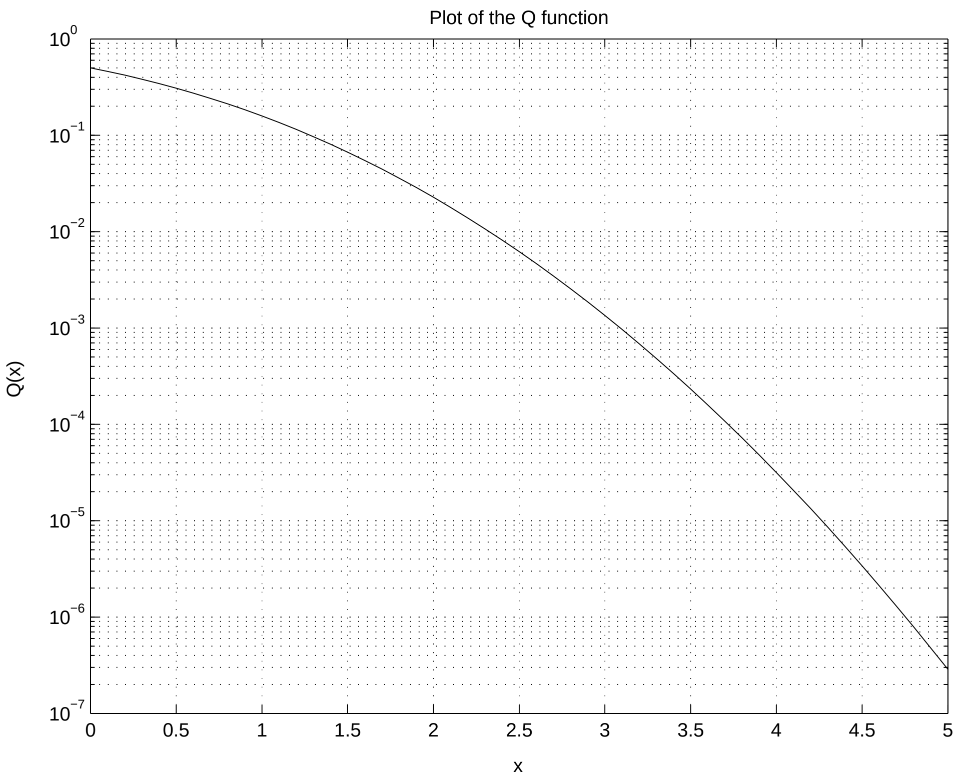


(a) For $\theta = 0$, find the probability of bit error for the upper branch and the lower branch. Evaluate for $A_c^2 = 300$, $T_b = 10^{-8}$, and $N_0 = 10^{-6}$

(b) For $\theta = -\pi/2$ find $r_1(nT_b)$ and $r_2(nT_b)$ for all 4 combinations of "1"s and "0"s that can be sent.

(c) For $\theta = -\pi/2$ find the probability of bit error for the upper branch and the lower branch.

This form of modulation is called Quadrature Phase Shift Keying, or QPSK. A form of QPSK, called offset QPSK, was used in many 2nd generation digital cellular phones, although most have migrated to other techniques.


Plot of the Q function
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
Q(x)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
x