EE 179: Introduction to Communications - Winter 2005

Final Exam

This exam is open book and notes, and any results derived in class, in the book, or in class handouts can be used without proof, as long as you state where the results come from. Sketches should indicate the general shape of the function and all key values on the x and y axes, including zero crossings. Good luck, and have a great break!

Problem 1 (35 points): Short Answer.

(a) What happens to the error probability of a BPSK signal if the data rate is increased and everything else (transmit power, noise PSD) stays the same?

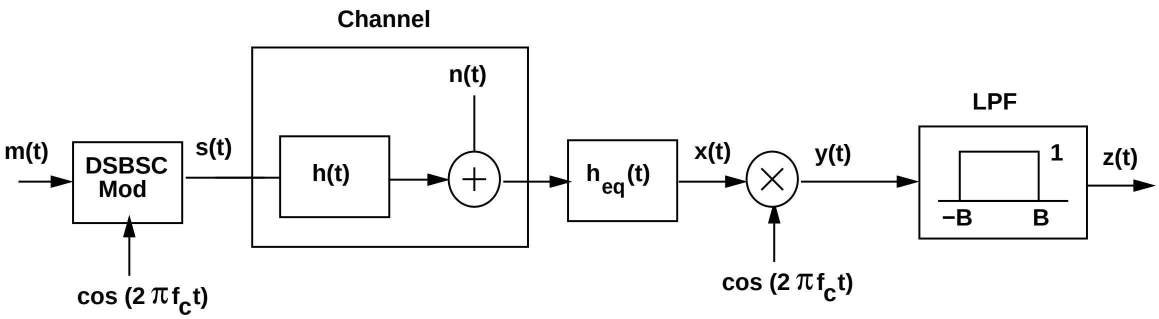
(b) Why would a standard AM signal received with an envelope detector sound better than one received with a product demodulator? What determines how bad the product demodulated signal would sound?

(c) Consider a zero-mean noise process $n(t)$ with autocorrelation $R_n(\tau) = cos(2\pi\tau/T)$. Find all values of Δt such that the samples $n(t)$ and $n(t+\Delta t)$ are uncorrelated. Let $m(t)$ be a baseband information signal with bandwidth B and power $P = 10$ W. Find the SNR of $m(t) + n(t)$ for $B > 1/T$ and $B < 1/T$.

(d) Let $n(t)$ be a WSS Gaussian process with mean $E[n(t)] = .5$ and autocorrelation $R_n(\tau) = .5N_0\delta(\tau)$. Find the distribution of $N = \int_0^T t^2 n(t)dt$.

Problem 2 (35 points): Equalization in AM Modulation.

Consider the DSBSC system shown below, where $s(t) = m(t)\cos(2\pi f_c t)$ with $f_c >> B$.



Assume the noise $n(t)$ has power spectral density $S_n(f) = .1$mW/Hz. The PSD $S_m(f)$ of $m(t)$ is defined as

$$S_m(f) = \begin{cases} 10 - 10|f|/B \text{ mW/Hz} & |f| \leq B \\ 0 & |f| > B \end{cases}$$

The frequency response of the channel $H(f)$ is defined as

$$H(f) = \begin{cases} 10 & |f - f_c| < .5B \\ .5 & .5B \leq |f - f_c| \leq B \\ 0 & \text{else} \end{cases}$$

(a) What is the power of $m(t)$?

(b) Sketch the PSD of the modulated signal $s(t)$

(c) Find the equalizer $H_{eq}(f)$ such that in the absence of noise (i.e. for $n(t) = 0$), $z(t) = m(t)$.

(d) Find the PSD and power of $z(t)$ due to noise only (i.e. for $m(t) = 0$), and due to signal only (i.e. for $n(t)$=0), assuming $H_{eq}(f)$ is the equalizer you found in part (c).

(e) Find the SNR of the receiver output with the equalizer of part (c).

Problem 3 (50 points): Analog Phase Modulation. *Note: There is a plot of Bessel function values $J_n(\beta)$, $n \leq 4$ on page 331 of the text. The plot only shows $J_n(\beta)$ for $n \geq 0$, values for $n < 0$ are obtained from the fact that $J_n(\beta) = -J_{-n}(\beta)$ for n odd and $J_n(\beta) = J_{-n}(\beta)$ for n even. Also assume throughout this problem that $J_n(\beta) = 0$ for $n > 4$ and any β, and that for $\beta \leq .3$, $J_0(\beta) = 1$, $J_1(\beta) = .5\beta = -J_{-1}(\beta)$, and $J_2(\beta) = J_3(\beta) = J_{-2}(\beta) = J_{-3}(\beta) = J_4(\beta) = J_{-4}(\beta) = 0$.*

Make sure your plot sketches below include all nonnegligible values.

Consider a phase-modulated (PM) signal $s(t) = 10\cos(2\pi f_c t + 2\pi k_p m(t))$, where $m(t)$ is an analog information signal.

(a) How would you generate $s(t)$ using an FM modulator (and possibly other components)? If $m(t)$ has bandwidth B, what is the bandwidth of the signal input to the FM modulator in your PM design?

(b) Find an approximation for the bandwidth of $s(t)$ based on Carson's rule and the relationship between phase and frequency modulation.

(c) Let $m(t) = 10\sin(2\pi f_m t)$. Let $f_m = 10$ KHz and $k_p = 2500/(2\pi f_m)$. Sketch $S(f)$ for the PM modulated signal $s(t)$. Find it's exact bandwidth based on your sketch, and its approximate value based on your approximation formula from part (b).

(d) Describe a method to demodulate $s(t)$ for $m(t)$ given in part (c), including the values for all parameters associated with your demodulator.

Problem 4 (40 points): BPSK Modulation with Biased Noise.

Consider a standard BPSK system with biased noise, so that the WSS noise random process $n(t)$ added in by the channel is Gaussian with PSD $S_n(f) = N_0/2$, but it has a nonzero mean $E[n(t)] = .5A_c \cos(2\pi f_c t)$. The input to the decision device over the kth bit time is still $r(kT_b) + n(kT_b)$, where

$$n(kT_b) = \int_0^{T_b} n(t) \cos(2\pi f_c t) dt.$$

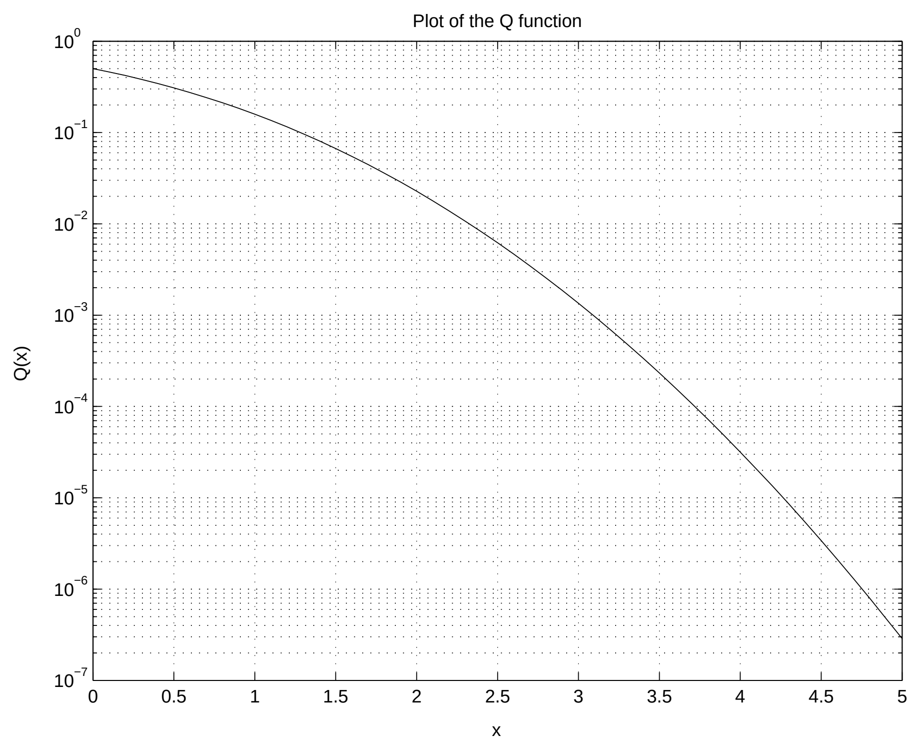
Assume the threshold T in the decision device does not change due to this bias, so $T = 0$. Also assume $f_c T_b >> 1$.

(a) Find the distribution of $n(nT_b)$.

(b) Find the probability of bit error in terms of E_b/N_0, assuming equally likely 1s and 0s are sent.

(c) Evalute P_b for $E_b/N_0 = 5$ dB. Compare this to the P_b with unbiased noise where $E[n(t)] = 0$.

The Q function is plotted below. Assume $Q(x) = 0$ for $x > 5$.

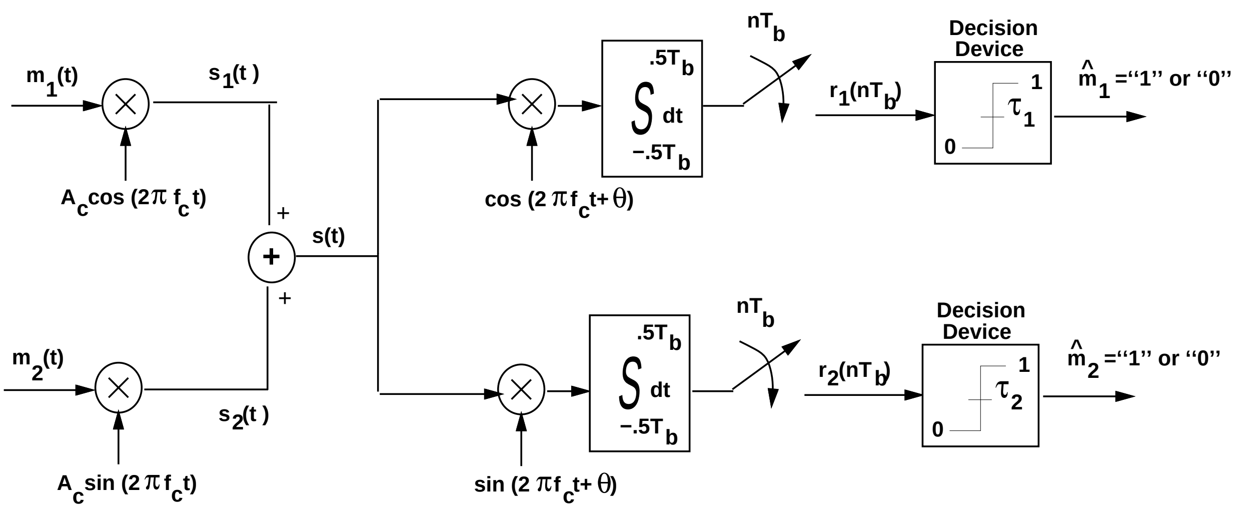


Problem 5 (40 points): Multiuser Modulation

We can simultaneously send information to two users by modulating the data associated with one user on a cosine carrier and that of the other user on a sine carrier, as illustrated in the figure below. Assume we use ASK modulation for each user. Thus, the signal $m_1(t)$ is a baseband on-off modulated signal associated with the bit stream of user 1, and the signal $m_2(t)$ is a baseband on-off modulated signal associated with the bit stream of user 2. (i.e. $m_i(t) = 1$ over the nth bit time if the nth bit is a "1" and $m_i(t) = 0$ over the nth bit time if the nth bit is a "0"). Thus, $s_1(t) = A_c m_1(t)\cos(2\pi f_c t)$ and $s_2(t) = A_c m_2(t)\sin(2\pi f_c t)$ are ASK modulated waveforms, where the carrier for $s_1(t)$ is $\cos(2\pi f_c t)$ and the carrier for $s_2(t)$ is $\sin(2\pi f_c t)$.

In the receiver the signal is split into an upper and a lower branch. The upper branch is multiplied by the cosine carrier with phase offset θ, integrated, and sampled to get $r_1(nT_b)$. The decision device in this branch has a threshold τ_1 such that for $r_1(nT_b) > \tau_1$, the bit corresponding to $m_1(nT_b)$ is decoded as a "1", and for $r_1(nT_b) \leq \tau_1$ this bit is decoded as a "0". A similar procedure follows in the lower branch: the signal is multiplied by the sine carrier with phase offset θ, integrated over a bit time, sampled, and passed through a decision device. This device decodes the bit corresponding to $m_2(nT_b)$ as a "1" if $r_2(nT_b) > \tau_2$ and as a "0" if $r_2(nT_b) \leq \tau_2$.

Assume throughout the problem that $f_c T_b >> 1$. Also, the trig identities $\cos a \cos b = .5[\cos(a+b) + \cos(a-b)]$, $\cos a \sin b = .5[\sin(a+b) - \sin(a-b)]$, and $\sin a \sin b = .5[\cos(a-b) - \cos(a+b)]$ may be useful.



(a) For $\theta = 0$, find $r_1(nT_b)$ and $r_2(nT_b)$ as a function of $m_1(nT_b)$ and $m_2(nT_b)$.

(b) For $\theta \sim \mathcal{U}[0, 2\pi]$, what should the threshold values τ_1 and τ_2 be and why (a qualitative answer is sufficient)?

(c) Suppose $\theta \neq 0$. The input to the decision device in the upper branch will then be $r_1(nT_b) + I_1(nT_b)$, where $I_1(nT_b)$ is interference caused by user 2's signal to user 1. Find $I_1(nT_b)$ as a function of θ and $m_2(nT_b)$.

(d) Using your answer to part (c), find the signal-to-interference (SIR) power ratio in the upper branch, defined as $E[r_1^2(nT_b)]/E[I_1^2(nT_b)]$, as a function of θ, assuming equally likely 1 and 0 bits for each user. For what values of θ does this SIR take on its maximum and minimum values?