EE 179: Introduction to Communications - Winter 2004

Final Exam

This exam is open book and notes, and any results derived in class, in the book, or in class handouts can be used without proof, as long as you state where the results come from. Sketches should indicate the general shape of the function and all key values on the x and y axes, including zero crossings. Graded exams and solutions will be available next week. Good luck, and have a great break!

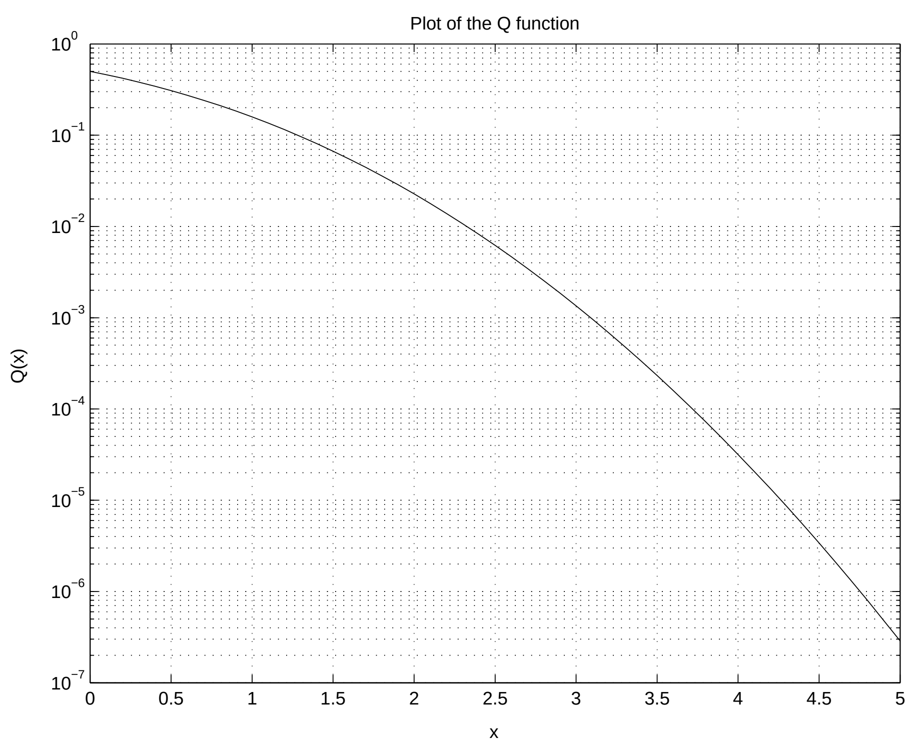
Problem 1 (40 points): Short Answer.

(a) Consider a random process $n(t)$ with PSD $S_n(f) = sinc(f)$. Find all values of Δt such that the samples $n(t)$ and $n(t + \Delta t)$ are uncorrelated?

(b) We discussed how an FM signal can be demodulated using either a differentiator followed by envelope detection or a zero-crossing detector. Discuss qualitatively what distortion each of these techniques introduces (i.e. the output signal for each techniques is $c_1\hat{m}(t) + c_2$ for constants c_1 and c_2 and due to distortion $\hat{m}(t) \neq m(t)$, why is $\hat{m}(t) \neq m(t)$ for each technique?).

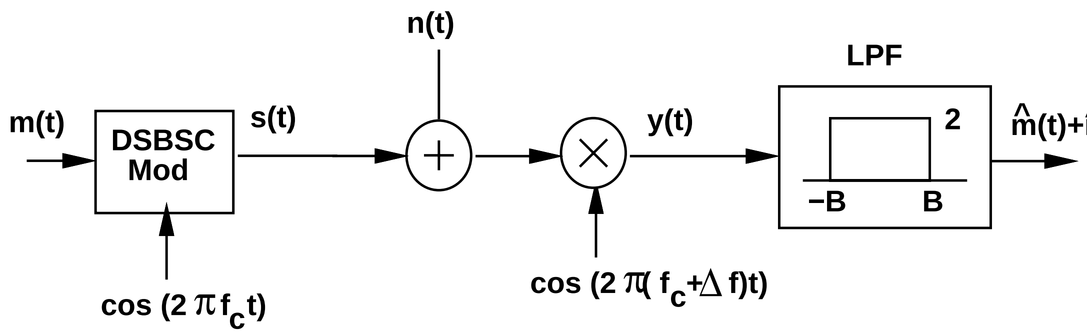
(c) Let $n(t)$ be a Gaussian process with mean zero and autocorrelation $R_n(\tau) = 1$. Find the distribution of $N = \int_0^T n(t)dt$.

(d) Assume a BPSK digital communication system with data rate 100 Kbps and additive white Gaussian noise (AWGN) with power spectral density $N_0 = 10^{-9}$ W/Hz. What transmit power is required for the transmitted signal $s(t) = \pm A_c \cos(2\pi f_c t)$ so that the bit error probability of this system is 10^{-3}. The Q function is plotted below.



Problem 2 (40 points): DSBSC with Frequency Offset.

Consider the DSBSC system shown below, where $f_c = 1$ GHz and $B = 30$ KHz. You can assume the oscillators in the transmitter and receiver are phase locked with a random phase that is uniformly distributed and independent of the noise.



The PSD $S_m(f)$ of $m(t)$ is given by

$$S_m(f) = \begin{cases} 1\text{mW/Hz} & |f| \leq .5B \\ 10\text{mW/Hz} & .5B < |f| \leq B \\ 0 & |f| > B \end{cases}$$

The noise is WSS with PSD

$$S_n(f) = \begin{cases} 1\text{mW/Hz} & |f| \leq f_c \\ 0 & |f| > f_c \end{cases}$$

The system output has two components: $\hat{m}(t)$ due to the information signal $m(t)$ and $\hat{n}(t)$ due to the noise $n(t)$

(a) For $\Delta f = 0$ find $S_{\hat{m}}(f)$, $S_{\hat{n}}(f)$, and the SNR at the system output.

(b) For $\Delta f = 35$ KHz, find $S_{\hat{m}}(f)$, $S_{\hat{n}}(f)$, and the SNR at the system output.

(c) Is SNR a good metric for performance for systems with frequency offset? Why or why not?

Problem 3 (30 points): FM Modulation. Consider an FM modulated signal

$$s(t) = 10 \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right]$$

Let $f_c = 10$ KHz, $k_f = 200$, and the information signal $m(t) = \sum_{n=-\infty}^{\infty} (-1)^n * 5rect(t - nT_0)$ for $T_0 = .001$.

(a) Find the bandwidth of $s(t)$ using Carson's rule and an appropriate bandwidth definition for $m(t)$.

(b) Sketch the instantaneous frequency $f_i(t)$ of $s(t)$.

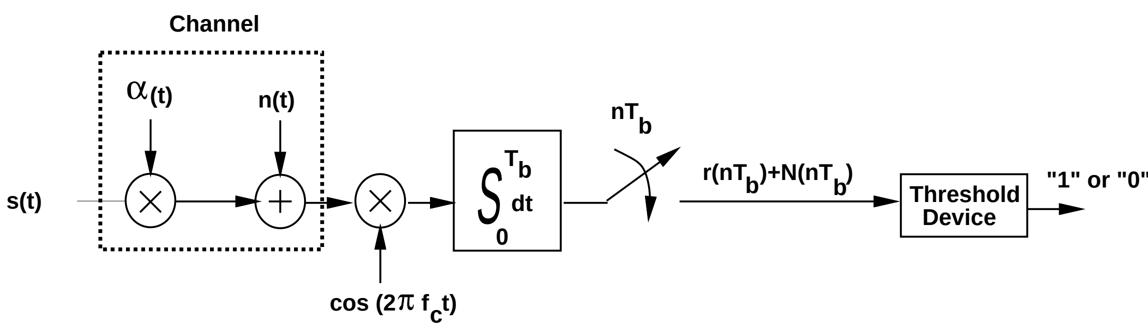
(c) Sketch $s(t)$.

(d) Assume this signal is demodulated with a zero-crossing detector. Find the minimum time interval T of the zero crossing detector such that there are at least 2 zero crossings in every interval. For this T, will there ever be more than 2 zero crossings?

Problem 4 (45 points): BPSK Modulation with Fading.

Wireless channels often introduce a random amplitude gain which affects probability of error in digital modulation. This problem explores the affect of this random amplitude gain, also called fading.

Consider the BPSK system shown below. The signal $s(t) = m(t)\cos(2\pi f_c t)$ is a BPSK modulated signal where $m(t)$ is a baseband polar modulated signal with amplitude A_c. The channel introduces both a random amplitude gain $\alpha(t)$ and additive white noise $n(t)$ with PSD $N_0/2$ The gain $\alpha(t) = \alpha(nT_b)$ is constant over a bit time and changes to another independent value at each bit time, with $p(\alpha(nT_b) = 1) = .8$ and $p(\alpha(nT_b) = .1) = .2$ on any given bit time. The input to the threshold device is $r(nT_b) + N(nT_b)$ where $N(nT_b)$ is based on $n(t)$ is not affected by the fading (so as derived in class, N is a Gaussian random variable with mean 0 and variance $.25N_0T_b$), and $r(nT_b)$ depends on both $s(t)$ and the random gain $\alpha(nT_b)$ over this bit time. In particular, given that a "1" (or a "0") is sent over a given bit time, $r(nT_b)$ is a random variable since it is a function of the random variable $\alpha(nT_b)$ over that bit time. The threshold device decides that a "1" was sent if its input is greater than zero, otherwise it decides that a "0" was sent. Assume $f_c >> 1$.



(a) Find the distribution of $r(nT_b)$ given that a "0" bit is transmitted.

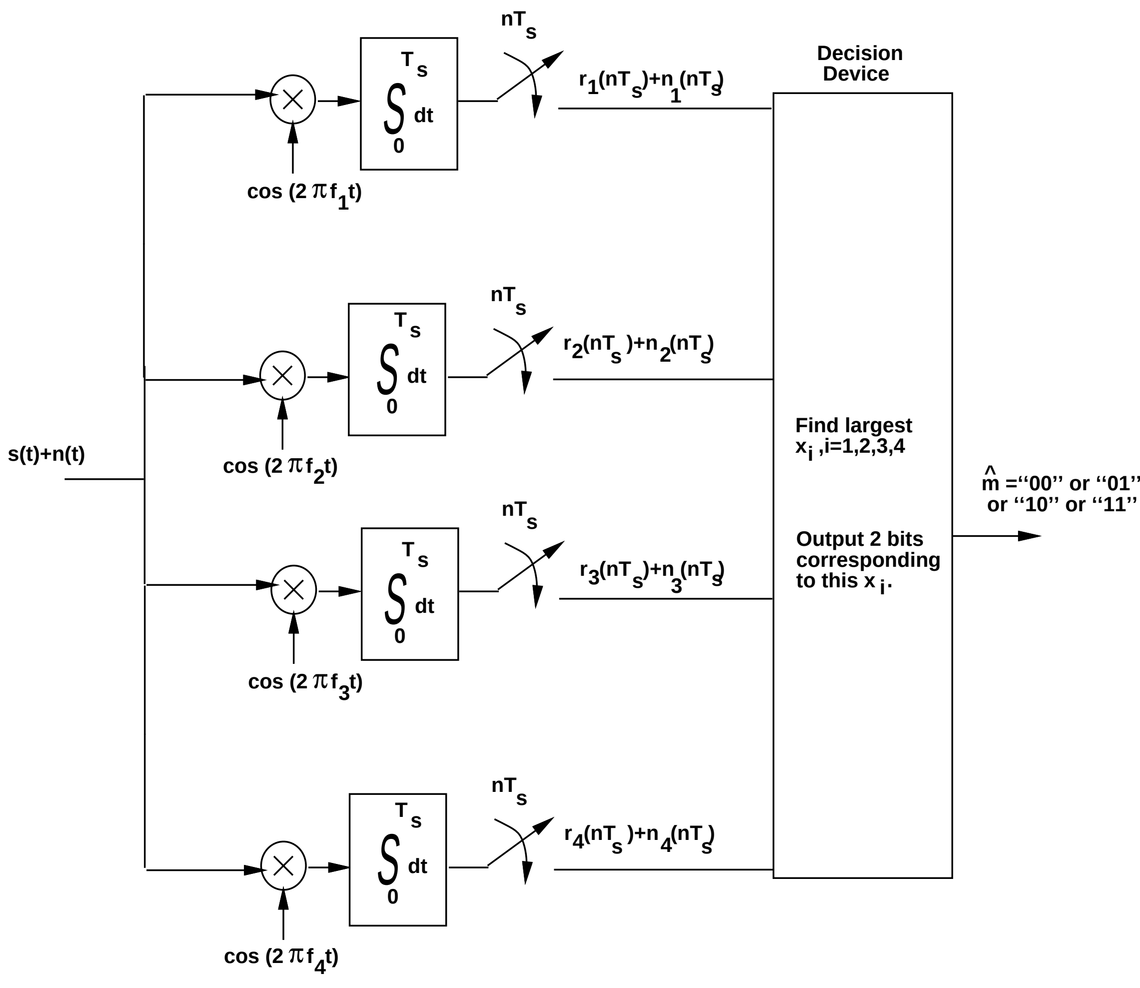
(b) Find the distribution of $r(nT_b)$ given that a "1" bit is transmitted.

(c) Find an expression for the probability of bit error for this system in terms of the Q function or complementary error function, A_c, T_b and N_0. You will need to condition on which bit was sent as well as on the value of α. Assume that "1" and "0" bits are equally likely to be transmitted.

(d) Evaluate P_b for $A_c = 2$ and $T_b/N_0 = 4$. Compare with P_b in the absence of fading (i.e. $\alpha(t) = 1$ for all t). (Note: there is a plot of the Q function on the first page of the exam).

Problem 5 (45 points): Multilevel FSK Modulation

In this problem we investigate 4 level FSK, which sends two bits per symbol time instead of one. Consider the FSK demodulator shown below, where $s(t) = A_c \cos(2\pi f_i t)$ for $f_i = f_1, f_2, f_3$ or f_4 over a symbol time T_s. The two bits per symbol time are encoded as "00″ $\rightarrow s(t) = A_c \cos(2\pi f_1 t)$, "01″ $\rightarrow s(t) = A_c \cos(2\pi f_2 t)$, "10″ $\rightarrow s(t) = A_c \cos(2\pi f_3 t)$, and "11″ $\rightarrow s(t) = A_c \cos(2\pi f_4 t)$. Assume the noise $n(t)$ is AWGN with mean zero and PSD $N_0/2$. The demodulator breaks the signal into 4 branches, with the input to the decision device corresponding to the ith branch given by $x_i = r_i(nT_s) + n_i(nT_s), i = 1, 2, 3, 4$. On the nth bit time the decision device in the demodulator outputs "00" if $x_1 > x_i, i = 2, 3, 4$, "01" if $x_2 > x_i, i = 1, 3, 4$, "10" if $x_3 > x_i, i = 1, 2, 4$, and "11" if $x_4 > x_i, i = 1, 2, 3$.



(a) What minimum frequency separation $|f_i - f_j|, i \neq j$ is needed for this system to work optimally. Explain what happens qualitatively if the frequency separation is less than this minimum value. Assume this minimum frequency separation for the remainder of the problem.

(b) Find the distribution of $n_i(nT_b), i = 1, 2, 3, 4$, and show that these 4 random variables are independent.

(c) Assuming the bits "00" (i.e. $s(t) = A_c \cos(2\pi f_1 t)$) are sent over a given symbol time, find $r_1(nT_s)$, $r_2(nT_s)$, $r_3(nT_s)$ and $r_4(nT_s)$.

(d) Find the probability of error given that "00" is sent in terms of the Q function or complementary error function, i.e. find $p(x_1(nT_s) < \max_{i=2,3,4} x_i(nT_s) | s(t) = A_c \cos(2\pi f_1 t))$.