CS 369 - Problem Set Two

Due Date: Thursday, May 23, 2013, by end of lecture

- Please make your answers clear and concise. In most cases, you should not need more than the equivalent of a page with size 12 font typed. You're more likely to get points subtracted if your proof is difficult to read or understand. To improve legibility, type your solutions if you can.
- Each solution should start on a new page.
- Searching for answers on the Web is not allowed.

1. **(10 points)** Consider load balancing on identical machines, where the jobs have *unknown durations*. Analyze the competitive ratio of the greedy algorithm in this case.
2. **(10 points)** Consider routing in the congestion-minimization model. Assume that all durations are multiples of q seconds, while arrivals are every second. Applying the approach discussed in class (and disregarding the "multiple of q" property) directly implies $O(\log nT)$-competitive algorithm. Can we get an $O(\log(nT/q))$-competitive algorithm ?
3. **(15 points)** In the online throughput maximization setting, consider the case where each request is of capacity (bandwidth requirement) 1 and each edge is of capacity 1 as well. Prove $\Omega(n)$ lower bound on any online algorithm in this setting.
4. **(20 points)** Consider congestion-minimization setting (see Section 7 in notes) where have added the notion of *cost* to every edge. First consider the case where the cost is equal to the relative load of an edge ℓ_e. In other words, if an edge is used to 70%, then it contributes 0.7 units to the total cost.

   We would like to prove that the cost achieved by the presented online algorithm is linear in the cost achieved by the optimum offline.

   (a) Let ℓ_e^* denote the optimum load on edge e after all requests were served; let ℓ_e denote the online load on edge e after all requests were served (by online). Review proof of Theorem 7.1 and show that

   $$\sum_{e \in E} (a^{\ell_e} - 1) \leq \gamma \sum_{e \in E} a^{\ell_e} \ell_e^*$$

   (b) Use part (a) together with the fact that for $0 < x < 1$ and $y \geq 0$ we have $1 + \frac{x}{2} y \leq (1+x)^y$ to prove the claim.

   (c) Consider an extension where the cost is computed as $cost(e)\ell_e$ where $cost(e)$ are non-negative integers. Show that for this case, (b) can be extended to clam that the algorithm is $O(\log \sum_{e \in E} cost(e))$-competitive with respect to congestion and within a constant factor of the cost achieved by the optimum.