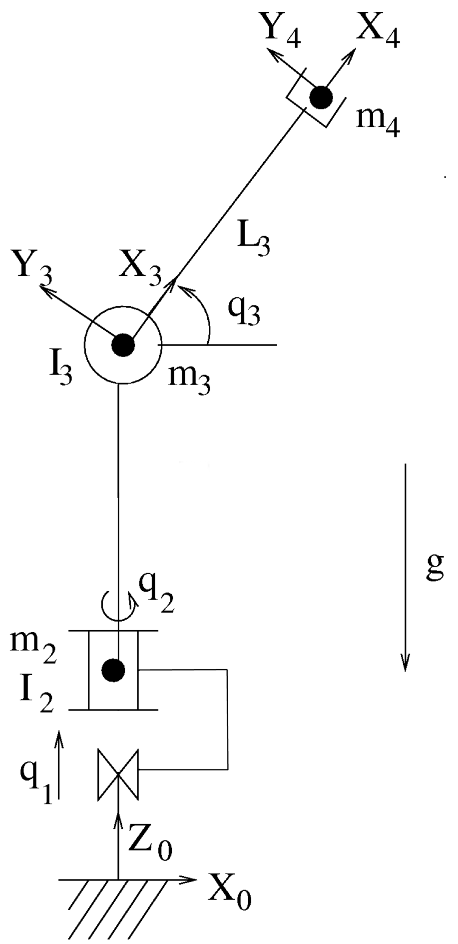
Advanced Robotic Manipulation Handout

CS327A (Spring 2011)

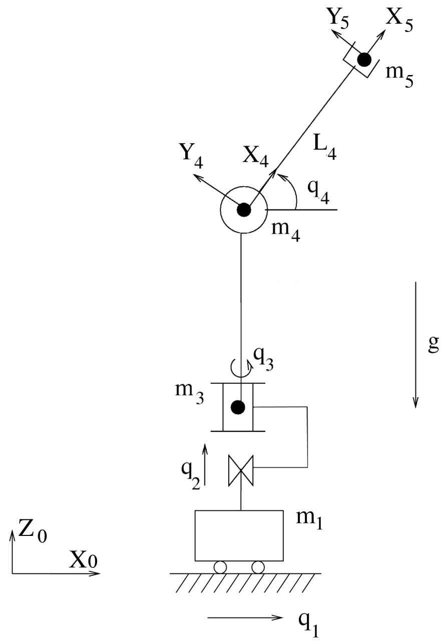
Problem Set #3 Due **Tue, May** 31^{st}

1. Consider the PRR manipulator given below.



Consider when $q_1 = 1m$ and $q_2 = 0°$ and $q_3 = 60°$, $L_3 = 1$, $m_i = 1$ for all i. Plot the translation *Belted Ellipsoid* in the X_3Y_3 plane (it will be helpful to use Matlab). What is the effective mass in $\widehat{X}_0$ direction?

2. Consider the PPRR macro/mini-manipulator shown below. This manipulator is built by combining a P manipulator and the PRR-spatial manipulator from Question 1. The task of this macro-/mini-manipulator is to *position* the end-effector. Notice that the numbering for the joint coordinates will now be different from Problem 1, since there is a new joint at the base. The configuration shown below is when $q_3 = 0$.



(a) Find the Jacobian matrix.

(b) Find the Jacobian matrix in frame $\{4\}$.

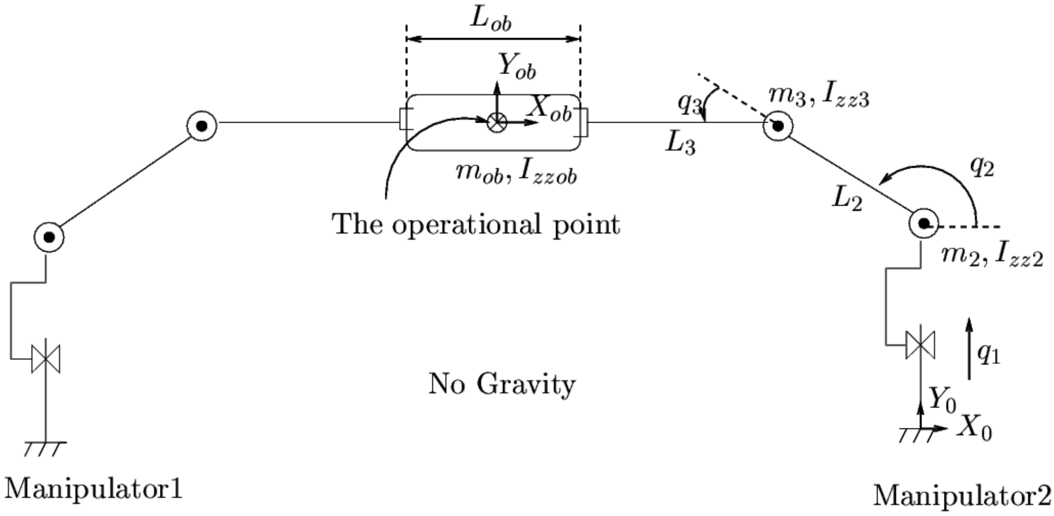
(c) Complete the joint space kinetic energy matrix $A(\mathbf{q})$. You may assume, as before, that $m_2 = 0$, ie. the vertical prismatic joint is massless. You may use solutions from Homework #3, but be careful with the change in joint numbering.

(d) Consider the configuration when $q_1 = 1m$, $q_2 = 1m$, $q_3 = 0°$, and $q_4 = 60°$. The masses are $m_i = 1kg$, the inertias are $I_{ii} = 0$, and $L_4 = 1m$. Plot the *Belted Ellipsoid* in X_4Y_4 plane. What is the effective mass in $\widehat{X}_0$ direction?

(e) Overplot your belted ellipsoid with that of Problem 1. Do your results satisfy the reduced effective mass property?

3. *Augmented Object Model/Virtual Linkage*

Let's consider two planar PRR-manipulators with an object. The task is to *position* and *orient* the object. Both PRR-manipulators are the same: $m_{object} = 1kg$, $I_{zz,object} = 1kgm^2$ and $L_{object} = 1m$.



The physical properties of each PRR-manipulator are $m_2 = m_3 = 1kg, I_{zz2} = I_{zz3} = 1kgm^2$, and $L_2 = L_3 = 1m$ (you may assume the prismatic joint is massless). The corresponding mass matrix and Jacobian for the end-effector are:

$$J = \begin{bmatrix} 0 & -L_2 sin(q_2) - L_3 sin(q_2 + q_3) & -L_3 sin(q_2 + q_3) \\ 1 & L_2 cos(q_2) + L_3 cos(q_2 + q_3) & L_3 cos(q_2 + q_3) \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -s_2 - s_{23} & -s_{23} \\ 1 & c_2 + c_{23} & c_{23} \\ 0 & 1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} m_2 + m_3 & m_3 L_2 c_2 & 0 \\ m_3 L_2 c_2 & m_3 {L_2}^2 + I_{zz2} + I_{zz3} & I_{zz3} \\ 0 & I_{zz3} & I_{zz3} \end{bmatrix} = \begin{bmatrix} 2 & c_2 & 0 \\ c_2 & 3 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

The Jacobian for the operational point is

$$J_{ob} = \begin{bmatrix} 0 & -L_2 sin(q_2) - (L_3 + \frac{L_{ob}}{2}) sin(q_2 + q_3) & -(L_3 + \frac{L_{ob}}{2}) sin(q_2 + q_3) \\ 1 & L_2 cos(q_2) + (L_3 + \frac{L_{ob}}{2}) cos(q_2 + q_3) & (L_3 + \frac{L_{ob}}{2}) cos(q_2 + q_3) \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -s_2 - 1.5 s_{23} & -1.5 s_{23} \\ 1 & c_2 + 1.5 c_{23} & 1.5 c_{23} \\ 0 & 1 & 1 \end{bmatrix}$$

Let's consider the configuration where $q_1 = 1m$, $q_2 = 30^o$, $q_3 = -30^o$ for the manipulator1 and $q_1 = 1m$, $q_2 = 150^o$, $q_3 = 30^o$ for the manipulator2.

(a) Calculate the pseudo kinetic energy matrix $\Lambda_{\oplus}$ for this augmented object.

(b) Find the W matrix, which relates resultant forces/moments and the applied forces/moments at the grasp points in the local frame of the object, i.e.

$$\begin{bmatrix} f_r \\ m_r \end{bmatrix} = \begin{bmatrix} f_{r,x} \\ f_{r,y} \\ m_r \end{bmatrix} = W \begin{bmatrix} f_{1,x} \\ f_{1,y} \\ f_{2,x} \\ f_{2,y} \\ m_1 \\ m_2 \end{bmatrix}, \text{where} \quad W = [W_f \; W_m].$$

(c) Find E matrix which relates the applied forces at the grasp points with internal forces, i.e.

$$\begin{bmatrix} f_{1,x} \\ f_{1,y} \\ f_{2,x} \\ f_{2,y} \end{bmatrix} = Et, \text{where } t \text{ is tension between two grasping points.}$$

(d) Compute the *Grasp Description Matrix*, G.