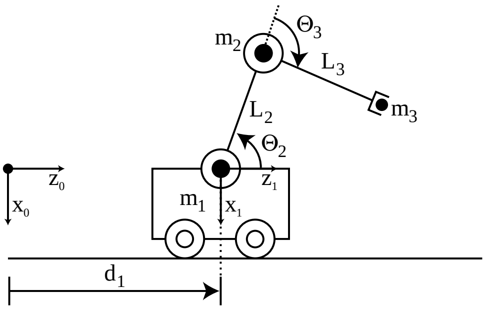
Advanced Robotic Manipulation — Handout

CS327A (Spring 2011)

Problem Set #2

Due **Wed, May** 4^{th}

1. [**20 marks**] Consider the PRR-planar mobile manipulator shown in the figure below. Assume that m_1, m_2, and m_3 are point masses.



   (a) Find the Jacobian matrix in frame $\{0\}$, relating velocity at the end-effector to the joint velocities. Use $[x \; z \; \alpha]^T$ as the operational coordinates where $\alpha = \theta_2 + \theta_3$.

   Hint: Use a geometry to find the Jacobian. In this case DH parameters might confuse you.

   (b) Find the singularities. Explain the physical meaning.

   (c) Derive the joint space kinetic energy matrix A and the gravity vector G.
   Use ${}^{C_1}I_1 = {}^{C_2}I_2 = {}^{C_3}I_3 = 0$.

   (d) Derive the operational space kinetic energy matrix Λ and the gravity vector P at the end-effector.

   (e) Consider the configuration when $\theta_2 = 45°$ and $\theta_3 = 90°$. Assume $d_1 = l_2 = l_3 = 1m$ and $m_1 = m_2 = m_3 = 1kg$ and the manipulator is at rest.

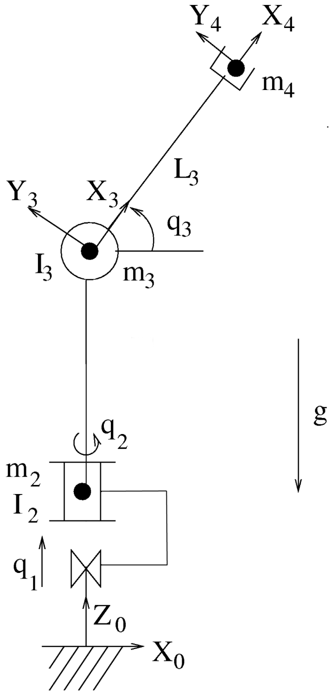
   Hint: At rest, centrifugal and coriolis force is zero.

      i. Find the joint torques required to compensate for gravity.

      ii. Determine the joint torques required to accelerate the base at $1m/s^2$ while maintaining the joint position of the arm. Use joint space equations of motion. Discuss your results.

      iii. Determine the joint torques required to accelerate the end-effector at $1m/s^2$ along the z_0-axis. Use operational space equations of motion. Discuss your results.

2. [**20 marks**] *Operational space* Consider the PRR-spatial manipulator shown in the figure below. Assume that m_4 is a point mass, $m_1 = 0$, and $I_i = 0$ for all joints. The task of this manipulator is to *position* the end-effector in the 3-D workspace. The configuration shown below is when $q_2 = 0$.



(a) Select a set of operational space coordinates for this task.

(b) Find the Jacobian matrix corresponding to the operational coordinates, in frame $\{0\}$.

(c) Find the Jacobian matrix corresponding to the operational coordinates, in frame $\{3\}$.

(d) Determine the kinematic singularities. Draw each one and explain its physical meaning.

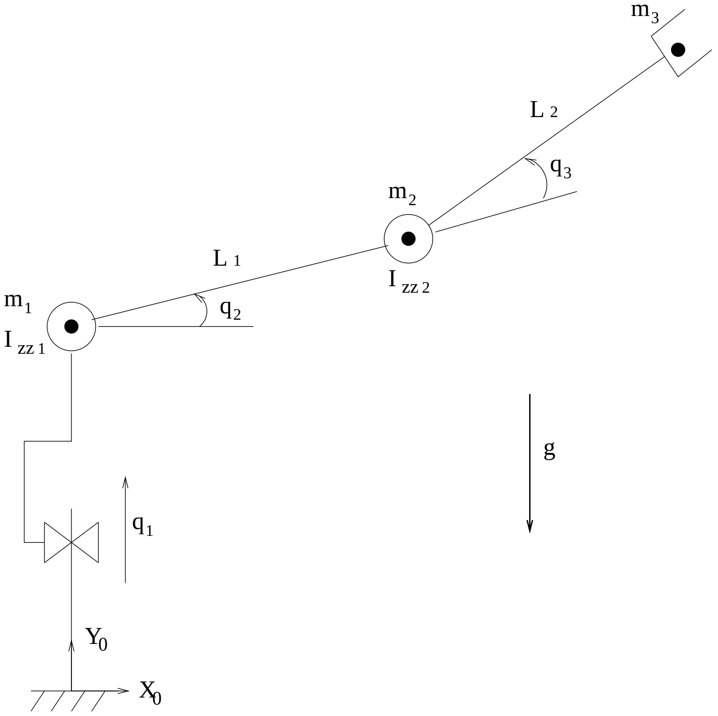
(e) Find the joint space kinetic energy matrix $A(\mathbf{q})$ and the joint space gravity vector $G(\mathbf{q})$.

(f) Find the configuration where the kinetic energy of the manipulator is minimum, for a given $\dot{\mathbf{q}}$ (you may assume $\dot{\mathbf{q}} \neq \mathbf{0}$).

(g) Take $m_2 = m_3 = 1kg$, and $L_3 = 1m$. Find the operational space kinetic energy matrix ${}^3\Lambda(\mathbf{q})$ and the operational space gravity vector ${}^3P(\mathbf{q})$ in frame $\{3\}$.

3. **[20 marks]** *Kinematics, Dynamics and Instantaneous Inverse Kinematics*

Consider the PRR planar manipulator shown in the figure below. The task of this manipulator is to *position* the end-effector. You are also given that $m_1 = m_2 = 1$, $L_1 = L_2 = 1$, $m_3 = 0$ and $I_{zz1} = I_{zz2} = 1$.



(a) Choose operational space coordinates appropriate for the task.

(b) Is this manipulator redundant with respect to the task? If so, what is the degree of the redundancy?

(c) Find the Jacobian matrix.

(d) Are there any singularities for this manipulator? If so, find the configurations from the Jacobian matrix and describe the resulting restriction in motion.

(e) For the configuration with $q_2 = 0°$ and $q_3 = -45°$, find the operational space kinetic energy matrix Λ.

Use $A = \begin{bmatrix} m_1 + m_2 & m_2 L_1 c_2 & 0 \\ m_2 L_1 c_2 & m_2 {L_1}^2 + I_{zz1} + I_{zz2} & I_{zz2} \\ 0 & I_{zz2} & I_{zz2} \end{bmatrix}$

(f) Consider another configuration: $q_2 = 90°$ and $q_3 = 0°$. For a given δx, we would like to find the joint angles δq. Determine δq using the pseudo inverse and the dynamically consistent inverse of J separately.

(g) When $\delta x = [0.1\ 0]^T$, calculate your δq for both cases and discuss the difference.

(h) What are the null spaces for both cases? Explain the possible motions without changing δx.

4. *Theory*

   (a) **[5 marks]** Why are 3 variables insufficient to describe rotations in 3-D space? Explain with examples and sketches.

   (b) **[5 marks]** Explain why dynamic decoupling is essential for operational space force control. *Hint : A few lines of text and/or a few equations will suffice.*

   (c) **[5 marks]** How will errors in the Kinematic model and the dynamic model affect an operational space controller's stability? In particular, what operational space parameters will they affect? Positions? Velocities? Accelerations? Stability?

   (d) **[10 marks]** What steps will you take to ensure that your controller is stable if you know that your kinematic and/or inertial model are/is flawed? *Hint : Think about how prevent injecting energy (ie. prevent making the system unstable), and how to handle singularities.*

   (e) **[15 marks]** Lagrange's method to determine a system's dynamics assumes that the trajectory a system takes optimizes the system's "action", which is a function of its generalized coordinates (q) and generalized velocities ($\dot{q}$). This assumption helps derive the Lagrangian dynamics formulation:

$$\tfrac{d}{dt}\left(\tfrac{\partial L(q,\dot{q})}{\partial \dot{q}}\right) - \tfrac{\partial L(q,\dot{q})}{\partial q} = 0$$

   In the course reader, we chose the Lagrangian to be $L(q,\dot{q}) = T(q,\dot{q}) - U(q)$, where T is the kinetic energy and U is the potential energy. Explain why this is an appropriate choice. *Hint : A possible line of argument could rely on other cases contradicting some laws of Physics. Another line of argument could be to use the initial assumption to justify the equations (doing the extra credit part might be quite helpful).*

   * **[Extra credit : 10 marks]** Starting with the initial assumption, pick an appropriate "action" and derive the Lagrangian dynamics formulation. *Hint : Google, Mechanics by Landau and Lifshitz.*