Advanced Robotic Manipulation

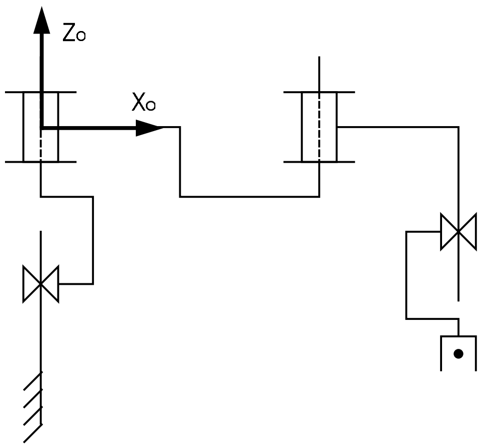
Handout

CS327A (Spring 2011)

Problem Set #1

Due **Monday, Apr 20**

1. [15 marks] **Jacobian Basics** : Consider the PRRP manipulator in the figure below.



For this manipulator, you are given the following homogeneous transformation matrices:

$$T_{01} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_{12} = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_{23} = \begin{bmatrix} c_3 & -s_3 & 0 & l_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_{34} = \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

(a) Give a set of generalized coordinates for the manipulator.

(b) Give a set of operational coordinates for the end-effector.

(c) How many d.o.f. does the manipulator have? How many d.o.f. does the end-effector have? What is the degree of redundancy of the manipulator?

(d) If the manipulator is to position an object in the space, what are your corresponding task configuration parameters? What is the degree of redundancy with respect to this task?

(e) If the manipulator is to orient an object, what are your corresponding task configuration parameters? What is the degree of redundancy with respect to this task?

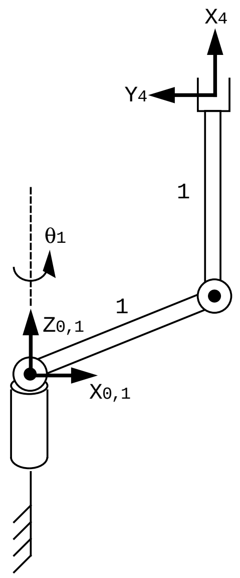
(f) Draw the frame {1} {2} {3} and {4}, then find the forward kinematics, T_{04}.

(g) Find the basic Jacobian matrix, $J_o(q)$.

(h) Find the Jacobian matrix, $J(q)$ for your operational space coordinate system choosen in (b).

(i) Find the corresponding E matrix such that $J(q) = E(X)J_o(q)$.

2. [25 marks] **Jacobian Without Rotation** : Consider the RRR-spatial manipulator in the figure below. For this manipulator, you are only to consider the *position* of the end-effector.



(a) Draw the frame {2} and {3}, then find the forward kinematics, T_{04}.

(b) Find the Jacobian matrix in frame {0}.

(c) Find the Jacobian matrix in frame {2}.

(d) Find the singularities of the Jacobian.

(e) Draw the robot configurations at the sigularities found in (d). What are the singular directions? Explain the physical meaning.

(f) Now, let's consider the configuration when $\theta_2 = 90°$ and $\theta_3 = 0°$. Here, the manipulator can be treated as a redundant manipulator in the subspace orthogonal to the sigular directions.

i. Find the Jacobian matrix in frame {2}. This matrix should be 1×3.

ii. Find the pseudo inverse of the Jacobian matrix.

iii. Find a matrix that will project a given vector to the associated null space.

iv. Describe this configuration. Discuss your results with respect to the general solution:

$$\delta q = J^{+}\delta x + [I_3 - J^{+}J]\delta q_0.$$

In particular, what are the resulting possible motions when you project an arbitrary vector δq_0 on to the null space?

3. [25 marks] **Jacobian Using Directional Cosines** : Consider the manipulator described by the following:

$$ {}^0_3T = \begin{bmatrix} 1 & 0 & 0 & d_1 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ {}^3_4T = \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S_4 & -C_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ {}^4_5T = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^5_6T = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S_6 & -C_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ and } {}^0_6T = \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & -C_4S_5 & d_1 \\ S_5C_6 & -S_5S_6 & C_5 & d_2 \\ -S_4C_5C_6 - C_4S_6 & S_4C_5S_6 - C_4C_6 & S_4S_5 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

(a) Draw a schematic of the robot, labeling all the generalized coordinates, and joint frames.

(b) Find the basic Jacobian matrix J_0.

(c) Determine the kinematic singularities and singular directions. Explain the physical meaning.

(d) Consider the representation $\mathbf{x} = [\mathbf{x}_p^T \ \mathbf{x}_r^T]^T$ consisting of the Cartesian coordinates for the position $\mathbf{x}_p$ of the wrist point and the direction cosines $\mathbf{x}_r$ for the orientation of the end-effector.

Find the Jacobian matrix J associated with this representation.

(e) Consider a starting configuration $\mathbf{q} = [1.0m \ 0.0m \ 0.5m \ 0° \ 90° \ 0°]^T$. and a desired end-effector goal configuration $\mathbf{x}_d = [\mathbf{x}_{pd}^T \ \mathbf{x}_{rd}^T]^T$ where $\mathbf{x}_{pd} = [0.9 \ \ 0.0 \ \ 0.6]^T$ and $\mathbf{x}_{rd} = [0.0 \ \ 1.0 \ \ 0.0 \ \ -0.12 \ \ 0.0 \ \ -0.99 \ \ -0.99 \ \ 0.0 \ \ 0.12]^T$.

Find the position error vector $\delta\mathbf{x}_p$ and the instantaneous angular error vector $\delta\Phi$.

4. [35 marks] **Designing a Robot** : Now that you know all about Jacobians for different coordinate representations, it is time to put that knowledge to use. For this question, you must design the kinematics for a robot that paints the inside surface of:

   (a) A cuboid : Dimensions 10m x 12m x 18m
   (b) A cylinder : Radius 7m, Height 15m
   (c) A spherical shell : Radius 8m.

   Assume that you have to place the robot inside each of the objects. Ie. The robot paints the object (all of the interior) from the inside. You can, however, pick where you want to place the robot.
   Your robot must have at least one revolute joint and may have zero or more prismatic joints. Assume that the robot's end effector (with the paintbrush) can paint horizontally and/or vertically.

   Answer these (separately for each object if needed):

   (a) How many degrees of freedom does your robot have? And why?
   (b) What joint types does your robot have? What are their joint-limits?
   (c) Where will you place your robot inside each object? How will you orient the objects?
   (d) Find the robot's basic Jacobian.
   (e) Assume that the robot's links are massless, and that the paintbrush weights 3kg. What is the maximum force or torque that each joint's motor must be capable of generating in order to guarantee that the robot can paint the objects? (Hint: $\Gamma = J^T F$. Also, is the robot's configuration singular?)
   (f) Now assume that each link is cylindrical and weighs 1kg/m along its height. Will the maximum required torques (or forces) increase or decrease? Explain for your robot, joint-by-joint.
   (g) Extra credit (10 marks) $= \frac{\text{Number of joints at which the torques/forces decrease}}{\text{Total number of joints}} * 10$

   Scoring details :

   (a) -5 for the second actuated dof, -10 for every next actuated dof. *Ie. 2 actuators = -5, 3 actuators = -15...*
   (b) +10 for the person with the lowest "maximum required torque", averaged across all joints (*compute this for your robot*).
   (c) The robot MUST be realistic. Ie. Given enough time, you should be able to build it. Also, describe all the assumptions you make.