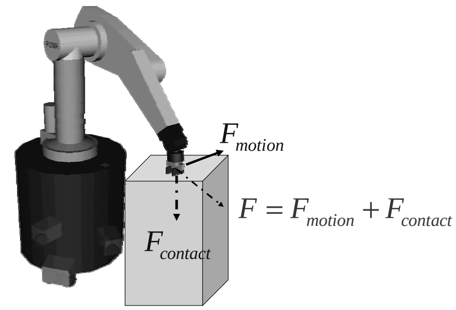
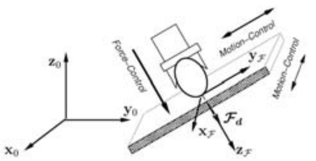
Task Control

Unified Motion/Force Control



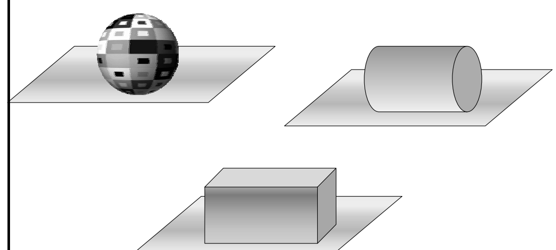
Unified Motion/Force Control



- Generalized Selection Matrix
- Dynamic Model (Homogeneity)

$$\Lambda_0(x)\,\dot{\vartheta} + \mu_o\left(x, \vartheta\right) + p_0(x) + F_{contact} = F_0$$

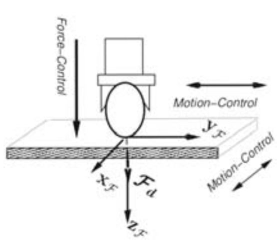
Task Description



Task Specification

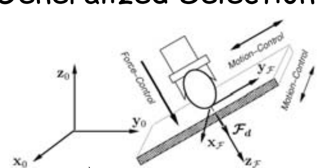
$$F = \Sigma F_{motion} + \bar{\Sigma} F_{force}$$

Selection matrix



$$\Sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \bar{\Sigma} = I - \Sigma$$

Generalized Selection Matrix



$$f_{(R_f)} = S_f\, f_{(R_0)}$$

Selection in R_f

S_f $\quad \Sigma \rightarrow \Omega$

$$\bar{\Sigma}\, f_{R_f}$$

Selection in R_0

$$\overbrace{S_f^T\, \bar{\Sigma}\, S_f}^{\bar{\Omega}}\, f_{R_0} \Leftarrow S_f^T\, \bar{\Sigma}\, f_{R_f}$$

Generalized Selection Matrix

$$\Sigma_F = \begin{pmatrix} \sigma_{F_x} & 0 & 0 \\ 0 & \sigma_{F_y} & 0 \\ 0 & 0 & \sigma_{F_z} \end{pmatrix}; \quad \bar{\Sigma}_F = I_3 - \Sigma_F$$

$$\Sigma_M = \begin{pmatrix} \sigma_{M_x} & 0 & 0 \\ 0 & \sigma_{M_y} & 0 \\ 0 & 0 & \sigma_{M_z} \end{pmatrix}; \quad \bar{\Sigma}_M = I_3 - \Sigma_M$$

Generalized Selection Matrix

$$\Omega = \begin{pmatrix} S_F^T \ \Sigma_F \ S_F & 0 \\ 0 & S_M^T \ \Sigma_M \ S_M \end{pmatrix}$$

$$\bar{\Omega} = \begin{pmatrix} S_F^T \ \bar{\Sigma}_F \ S_F & 0 \\ 0 & S_M^T \ \bar{\Sigma}_M \ S_M \end{pmatrix}$$

Basic Dynamic Model

Operational force $\stackrel{?}{\equiv}$ Forces & Moments

$$\dot{X} = J(q)\dot{q}$$

$$\Gamma = J^T(q)F$$

Forces & Moments

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = J_0(q)\dot{q}$$

$$J\dot{q} = E(X)J_0\dot{q}$$

$$\dot{X} = E\begin{pmatrix} v \\ \omega \end{pmatrix}$$

$$F_0 \doteq \begin{pmatrix} f \\ m \end{pmatrix};$$

$$J^T F = J_0^T E^T F$$

$$\begin{pmatrix} f \\ m \end{pmatrix} = E^T \ F$$

Basic Dynamic Model

$$\Lambda_0 = E^T \Lambda E$$

$$\Lambda \ddot{x} + \mu(x, \dot{x}) + p(x) = F$$

$$\Downarrow E^T$$

$$\boxed{\Lambda_0 \dot{\vartheta} + \mu_0(x, \vartheta) + p_0(x) = F_0}$$

with $\vartheta \mathrel{\hat{=}} \begin{pmatrix} v \\ \omega \end{pmatrix}$

Orientation Representation

$$x_r, \ x_{rd} \rightarrow \delta x_r = x_r - x_{rd}$$

$$\dot{x}_r = E_r \ \omega$$

Instantaneous Angular Error

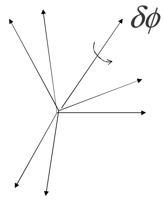
$$\delta x_r = E_r \ \delta\phi$$

Instantaneous Angular Error

$$\dot{x}_r = E_r \omega$$

$$\delta x_r = E_r \, \delta\phi$$

$$\boxed{\delta\phi = E_r^+ \, \delta x_r}$$



Control – Position Errors

$$\left(x - x_d\right) = \begin{pmatrix} x_p - x_{pd} \\ x_r - x_{rd} \end{pmatrix}$$

$$\dot{x}_r = E_r \, \omega$$

$$\underline{\delta x_r} = E_r \, \underline{\delta\phi} \quad \Rightarrow \quad \boxed{\begin{pmatrix} x_p - x_{pd} \\ \delta\phi \end{pmatrix} \text{ Error Vector}}$$

$$\delta x_r = \left(x_r - x_{r(d)}\right) = E_r \, \delta\phi$$

$$\boxed{\delta\phi = E_r^+ \left(x_r - x_{r(d)}\right)}$$

Goal Position

$$x_d = \begin{bmatrix} x_{pd} \\ x_{rd} \end{bmatrix}$$

$$f^* = -k_p\left(x_p - x_{pd}\right) - k_v \dot{x}_p$$

$$m^* = -k_p \delta\phi - k_v \omega$$

with $\delta\phi = E_r^+\left(x_r\right)\left(x_r - x_{rd}\right)$

Closed loop

$$I.\ddot{x}_p + k_v \dot{x}_p + k_p\left(x_p - x_{pd}\right) = 0$$

$$I\,\dot{\omega} + k_v \omega + k_p \delta\phi = 0$$

Direction Cosines

$$\mathbf{x}_r = \left(S_1^T S_2^T S_3^T\right)^T$$

$$\mathbf{x}_{rd} = \left(S_{1d}^T S_{2d}^T S_{3d}^T\right)^T$$

$$E_r^+ = \frac{1}{2} E_r^T$$

where $E_r^T(\mathbf{x}_r) = \left(-\hat{S}_1^T \quad -\hat{S}_2^T \quad -\hat{S}_3^T\right)$

$$\delta\Phi = \frac{1}{2} E_r^+(\mathbf{x}_r)(\mathbf{x}_r - \mathbf{x}_{rd})$$

$$\delta X_r = \begin{bmatrix} S_1 - S_{1d} \\ S_2 - S_{2d} \\ S_3 - S_{3d} \end{bmatrix} = \begin{bmatrix} S_1 \\ S_2 \\ S_1 \end{bmatrix} - \begin{bmatrix} S_{1d} \\ S_{2d} \\ S_{3d} \end{bmatrix}$$

since

$$E_r^T(\mathbf{x}_r)\mathbf{x}_r = \hat{S}_1 S_1 + \hat{S}_2 S_2 + \hat{S}_3 S_3 = 0$$

The angular rotation error

$$\delta\Phi = -\frac{1}{2}\left(\hat{S}_1 S_{1d} + \hat{S}_2 S_{2d} + \hat{S}_3 S_{3d}\right)$$

Euler Parameters

The end-effector orientation

$$\mathbf{x}_r = \lambda = \left(\lambda_0 \lambda_1 \lambda_2 \lambda_3\right)^T$$

The desired orientation

$$\lambda_d = \left(\lambda_{0d} \lambda_{1d} \lambda_{2d} \lambda_{3d}\right)^T$$

The orientation error vector

$$\delta\lambda = \lambda - \lambda_d$$

The left inverse

$$E_r^+(\mathbf{x}_r) = 2\overset{\vee}{\lambda}^T$$

where

$$\overset{\vee}{\lambda}^T = \begin{pmatrix} -\lambda_1 & \lambda_0 & -\lambda_3 & \lambda_2 \\ -\lambda_2 & \lambda_3 & \lambda_0 & -\lambda_1 \\ -\lambda_3 & -\lambda_2 & \lambda_1 & \lambda_0 \end{pmatrix}$$

Noting that

$$\overset{\vee}{\lambda}^T \lambda = 0$$

The angular rotation error

$$\boxed{\delta\Phi = -2\overset{\vee}{\lambda}^T \lambda_d}$$

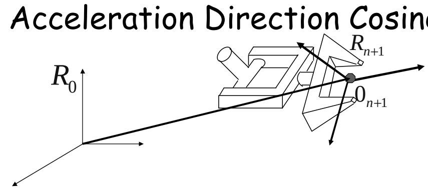
Angular Acceleration

$$\dot{X}_r = E_r\omega$$

$$\ddot{X}_r = E_r\dot{\omega} + \dot{E}_r\omega$$

$$\boxed{\dot{\omega} = E_r^+\ddot{X}_r - E_r^+\dot{E}_r\omega}$$

Acceleration Direction Cosines



The orientation is described by

$$\mathbf{x}_r = \left(S_1^T S_2^T S_3^T\right)^T$$

$$S1 = x_{(n+1)};\ S2 = y_{(n+1)};\ S3 = z_{(n+1)};$$

The second time derivatives

$$\frac{d^2\mathbf{x}_{(n+1)}}{dt^2} = -\mathbf{x}_{(n+1)}\times\dot{\omega} + \left(\mathbf{x}_{(n+1)}\times\omega\right)\times\omega$$

$$\frac{d^2\mathbf{y}_{(n+1)}}{dt^2} = -\mathbf{y}_{(n+1)}\times\dot{\omega} + \left(\mathbf{y}_{(n+1)}\times\omega\right)\times\omega$$

$$\frac{d^2\mathbf{z}_{(n+1)}}{dt^2} = -\mathbf{z}_{(n+1)}\times\dot{\omega} + \left(\mathbf{z}_{(n+1)}\times\omega\right)\times\omega$$

However

$$\mathbf{u}\times\mathbf{v}\times\mathbf{w} = \left(\mathbf{u}^T\mathbf{v}\right)\mathbf{w} - \left(\mathbf{v}^T\mathbf{w}\right)\mathbf{u}$$

This yields

$$\ddot{\mathbf{x}}_r = E(\mathbf{x}_r)\dot{\omega} + R(\mathbf{x}_r,\omega)\omega - (\omega^T\omega)\mathbf{x}_r$$

where

$$R(\mathbf{x}_r,\omega) = \begin{pmatrix} \left(S_1^T\omega\right)I_3 \\ \left(S_2^T\omega\right)I_3 \\ \left(S_3^T\omega\right)I_3 \end{pmatrix}$$

Acceleration Direction Cosines

$$\dot{\omega}_d = \frac{1}{2} E^T \ddot{x}_{rd} + \frac{1}{2} R^T(x_r, \omega)\dot{x}_{rd}$$

Euler Parameters

The acceleration associated with Euler parameters

$$\ddot{\lambda} = \frac{1}{4}\check{\lambda}\,\dot{\omega} - \frac{1}{2}(\omega^T \omega)\lambda$$

since $\quad \check{\lambda}^T \lambda = 0$

Euler Parameters

The angular acceleration vector

$$\dot{\omega} = 4\check{\lambda}^T \ddot{\lambda}$$

The desired angular acceleration

$$\dot{\omega}_d = 4\check{\lambda}_d^T \ddot{\lambda}_d$$

Motion Tracking $(x_{pd}\ ,\ \dot{x}_{pd}\ ,\ \ddot{x}_{pd})$

$$F^* = \ddot{x}_{pd} - k_p\left(x_p - x_{pd}\right) - k_v\left(\dot{x}_p - \dot{x}_{pd}\right)$$

Closed loop

$$I.\ddot{\varepsilon}_x + k_v\dot{\varepsilon}_x + k_p\varepsilon_x = 0$$

with

$$\varepsilon_{x_p} = x_p - x_{pd}$$

Motion Tracking $(x_{pd}\ ,\ \dot{x}_{pd}\ ,\ \ddot{x}_{pd})$

$$f^* = \ddot{x}_{pd} - k_p\left(x_p - x_{pd}\right) - k_v\left(\dot{x}_p - \dot{x}_{pd}\right)$$

$$m^* = \dot{\omega}_d - k_p\delta\phi - k_v\left(\omega - \omega_d\right)$$

with $\quad \delta\phi = E_r^+\left(x_r - x_{rd}\right)$

$$\omega_d = E_r^+\left(x_{rd}\right)\dot{x}_{rd}$$

and

$$\dot{\omega}_d = E_r^+\left(x_{rd}\right)\ddot{x}_{rd} - E_r^+\left(x_{rd}\right)\dot{E}\left(x_{rd}\right)\omega_d$$

Closed loop

$$\left(\ddot{x}_p - \ddot{x}_{pd}\right) + k_v\left(\dot{x}_p - \dot{x}_{pd}\right) + k_p\left(x_p - x_{pd}\right) = 0$$

$$\left(\dot{\omega} - \dot{\omega}_d\right) + k_v\left(\omega - \omega_d\right) + k_p\delta\phi = 0$$

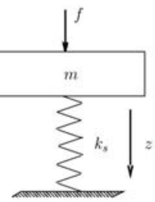
A Mass Spring System

System

$$m\ddot{z} + k_s z = f$$

$$f_s = k_s z$$

$$m\frac{1}{k_s}\ddot{f}_s + f_s = f$$



System $m\frac{1}{k_s}\ddot{f}_s + f_s = f$

Control

$$f = f_s + m \bullet f_{comp}$$

$$f = f_s - m\left[k_f\left(f_s - f_d\right) + k_{v_f}\dot{f}_s\right]$$

Control-loop System

$$\ddot{f}_s + k_s k_{v_f}\dot{f}_s + k_s k_f\left(f_s - f_d\right) = 0$$

Static Equilibrium

$$f_s = f_d$$

End-Effector/Sensor System

$$\Lambda_0\dot{\vartheta} + \mu_0(x,\vartheta) + p_0(x) + F_{contact} = F_0$$

Unified Control

$$F_0 = F_{motion} + F_{force}$$

$$F_{motion} = \hat{\Lambda}_0\,\Omega\,F^*_{motion} + \hat{\mu}_0 + \hat{P}_0$$

$$F_{force} = \hat{\Lambda}_0\,\bar{\Omega}\,F^*_{force} + F_{sensor}$$

End-Effector/Sensor System

$$\Lambda_0\dot{\vartheta} + \mu_0(x,\vartheta) + p_0(x) + F_{contact} = F_0$$

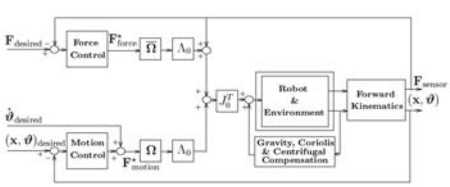
Unified Control

$$F_0 = F_{motion} + F_{force}$$

$$F_{motion} = \hat{\Lambda}_0\,\Omega\,F^*_{motion} + \hat{\mu}_0 + \hat{P}_0$$

$$F_{force} = \hat{\Lambda}_0\,\bar{\Omega}\,F^*_{force} + \bar{\Omega}\,F_{desired}$$

Unified Motion & Force Control



Two decoupled Subsystems

$$\Omega\,\dot{\vartheta} = \Omega\,F^*_{motion}$$

$$\bar{\Omega}\,\dot{\vartheta} = \bar{\Omega}F^*_{force}$$