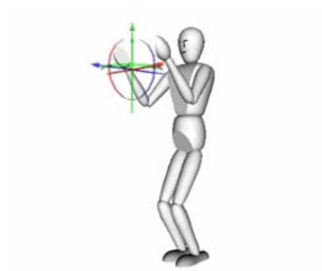
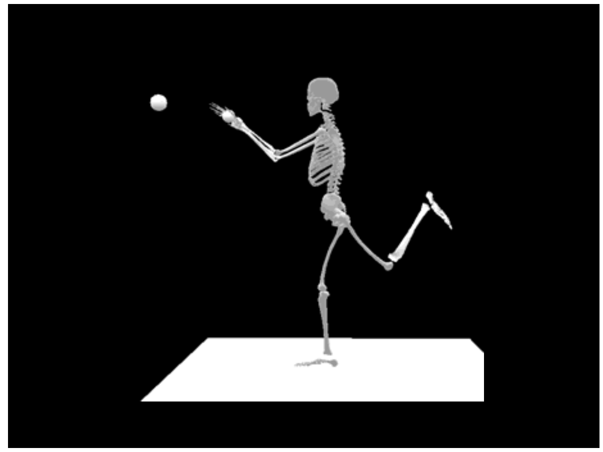
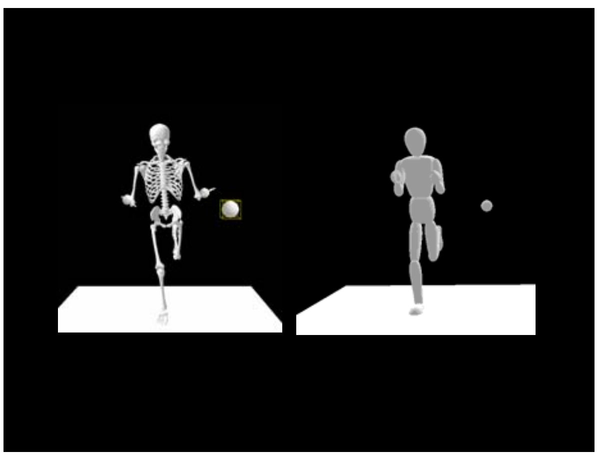


Redundancy





Equations of Motion

Joint Space

$$A(q)\ddot{q} + b(q,\dot{q}) + g(q) = \Gamma$$

Operational Space

$$\Lambda(x)\ddot{x} + \mu(x,\dot{x}) + p(x) = F$$

Relationships

$$\Gamma = J^T F$$

$$(A\ddot{q} + b + g) = J^T(\Lambda\ddot{x} + \mu + p)$$

$$(A\ddot{q} + b) = J^T(\Lambda\ddot{x} + \mu)$$ Inertial forces

Non Redundancy

$A\ddot{q} + b + g = \Gamma$ (joint dynamics)

J^{-T} J^T

$\Lambda\ddot{x} + \mu + p = F$ (Task dynamics)

Redundancy

$A\ddot{q} + b + g = \Gamma$ (joint dynamics)

projection $\bar{J}^T$ J^T

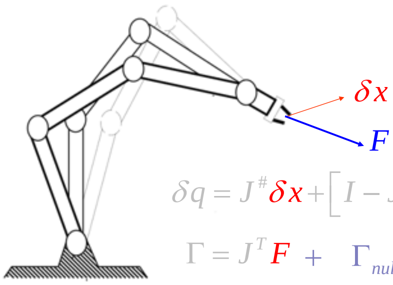
$\Lambda\ddot{x} + \mu + p = F$ (Task dynamics)

where

$\bar{J} = A^{-1}J^T\Lambda$ and $\Lambda^{-1} = J A^{-1} J^T$

$\bar{J}$: dynamically consistent generalized inverse

Redundancy



$$\delta q = J^{\#}\delta x + \left[I - J^{\#}J\right]\delta q_0$$

$$\Gamma = J^T F \;+\; \Gamma_{nullspace}$$

Redundancy

Joint/Task Displacements

$$\delta x = J\,\delta q$$

$$\delta q = J^{\#}\delta x + \left[I - J^{\#}J\right]\delta q_0$$

Joint/Task Forces

$$\Gamma = J^T F$$

$$F = ?$$

$$\Gamma = J^T F$$

Given F , Γ is $\left(J^T F\right)$

Given Γ , what is F

$$F = J^{\#^T}\Gamma \;?$$

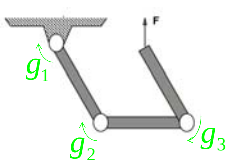
However

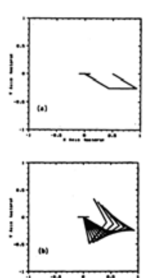
different selections of $J^{\#}$ $\left(J = J\,J^{\#}J\right)$

would lead

to different solutions

Gravity Example

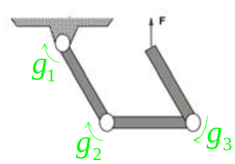


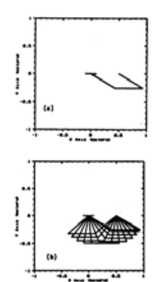


$p = ?$ $\quad p = J^{+^T} g$

$$J^+ = J^T\left(J\,J^T\right)^{-1}$$

Gravity Example

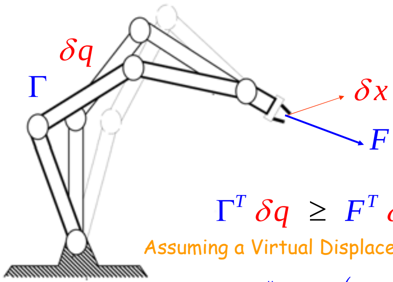




$p = ?$ $\quad p = \bar{J}^T g$

$$\bar{J} = A^{-1} J^T \Lambda$$

Redundancy



$$\Gamma^T \, \delta q \;\geq\; F^T \, \delta x$$

Assuming a Virtual Displacement

$$\delta q = J^{\#} \, \delta x + \left(I - J^{\#} J\right) \delta q_0$$

Virtual Displacement $\quad \delta q = J^{\#} \, \delta x + \left(I - J^{\#} J\right) \delta q_0$

Virtual Work $\quad \delta w = \Gamma^T \, \delta q$

$$\delta w = \delta w_1 + \delta w_2$$

$$\left(J^{\#^T} \Gamma\right)^T \delta x \qquad \left[\left(I - J^{\#} J\right)^T \Gamma\right]^T \delta q_0$$

$$\Downarrow$$

$$\Gamma = J^T \left(J^{\#^T} \Gamma\right) + \left(I - J^T J^{\#^T}\right) \Gamma$$

Decomposition

$$\Gamma = J^T \left[J^{\#^T} \Gamma\right] + \left[I - J^T J^{\#^T}\right] \Gamma$$

Task Space Forces (F)

Joint Torques acting in the null space

$$\Gamma = J^T F + \left[I - J^T J^{\#^T}\right] \Gamma_0$$

Dynamic Constraints

$$A(q)\,\ddot{q} + b(q,\dot{q}) + g(q) = \Gamma$$

$$\Gamma = J^T F + \left[I - J^T J^{\#^T}\right] \Gamma_0$$

$$A\ddot{q} + (b + g) = J^T F + \left[I - J^T J^{\#T}\right] \Gamma_0$$

$$\ddot{q} + A^{-1}(b+g) = A^{-1} J^T F + A^{-1}\left[I - J^T J^{\#T}\right] \Gamma_0$$

$$J \quad \Downarrow$$

$$J\ddot{q} + JA^{-1}(b+g) = JA^{-1} J^T F + JA^{-1}\left[I - J^T J^{\#T}\right] \Gamma_0$$

$$J\ddot{q} = \ddot{x} - \dot{J}\dot{q} \quad \Downarrow$$

$$\ddot{x} + \left[JA^{-1}(b+g) - \dot{J}\dot{q}\right] = \left(JA^{-1} J^T\right) F + JA^{-1}\left[I - J^T J^{\#T}\right] \Gamma_0$$

$$\Lambda^{-1} \qquad \ddot{x}_n = 0$$

Dynamic Consistency

$\Gamma \longrightarrow J A^{-1} \Gamma$

joint torques $\qquad$ task acceleration

Relationship

$$\Gamma = J^T F + \left(I - J^T J^{\#^T}\right) \Gamma_0$$

Dynamic Constraint

$$JA^{-1}\left(I - J^T J^{\#^T}\right) \Gamma_0 \equiv 0$$

$$JA^{-1} = \left(JA^{-1} J^T\right) J^{\#^T}$$

$$\Lambda \leftarrow \left(JA^{-1} J^T\right)^{-1} JA^{-1} = J^{\#^T}$$

Dynamic Consistency

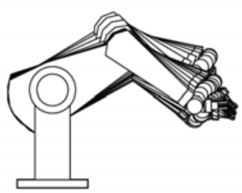
$\bar{J}(q)$ is the Dynamically Consistent Generalized Inverse

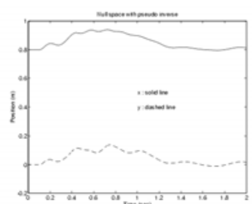
Theorem (Consistency)

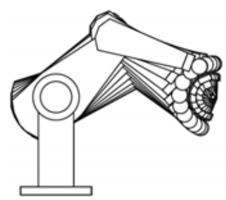
$\bar{J}$ is unique and $\bar{J} = A^{-1} J^T \Lambda$

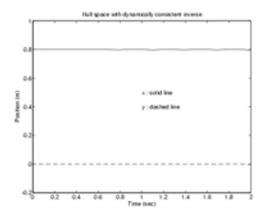
Non-redundant

$$\bar{J} = J^{-1}$$









Velocity Force Duality

	Velocity	Force
Non Red.	$\delta q = J^{-1} \delta x$	$\Gamma = J^T F$
Redundant	$\delta q = \bar{J} \delta x + \left[I - \bar{J} J \right] \delta q_0$	$\Gamma = J^T F + \left[I - J^T \bar{J}^T \right] \Gamma_0$

Task dynamics

$$\Lambda(q)\ddot{x} + \mu(q,\dot{q}) + p(q) = F$$

$$\Lambda = \left(J A^{-1} J^T \right)^{-1}$$

$$\mu(q,\dot{q}) = \bar{J}^T b(q,\dot{q}) - \Lambda(q) \dot{J}(q) \dot{q}$$

$$p(q) = \bar{J}^T g(q)$$

Redundant Robot Control

Task Space: J^T

Null Space: N^T where $N = I - \bar{J}J$

Robot Control

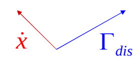
$$\Gamma = J^T F + N^T \Gamma_0$$

Γ_1 Γ_2

dynamically decoupled

Stability

$\Gamma_{dis}^T \dot{q} \leq 0$; for $\dot{q} \neq 0$



$$\Gamma_{dis} = -k_v J^T \dot{x} = -k_v J^T J \dot{q}$$

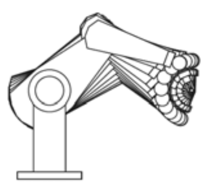
$$\dot{q}^T D(q) \dot{q} \leq 0 ; \qquad \dot{q} \neq 0$$

$$D(q) = k_v \left(J^T J \right)$$

$J^T J$: is a n x n matrix of rank m_0
it is Positive Semi-definite

The System is Stable, but not asymptotically stable

$$\dot{q}^T D(q) \dot{q} = 0$$



Asymptotic Stability

$\Gamma_{dis}^T \dot{q} < 0$; for $\dot{q} \neq 0$

$$\Gamma_{dis} = -k_v J^T J \dot{q} - k_{vq} \dot{q}$$

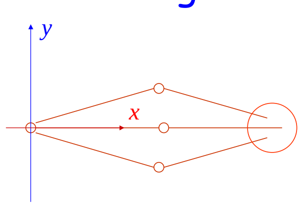
$$\Downarrow$$

$$D(q) = k_v J^T J + k_{vq} I_n$$

Positive definite

$\dot{q}^T D(q) \dot{q} < 0$ for $\dot{q} \neq 0$

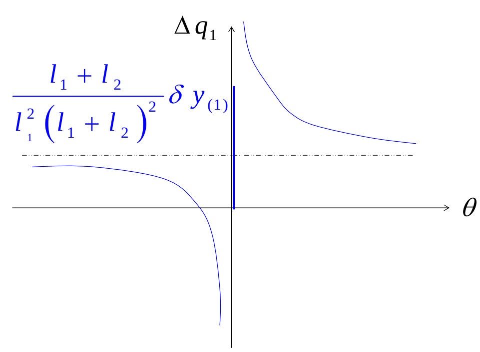
Kinematic Singularities



Joint Space Formulation

Find a pseudo-inverse J^+

Pseudo Inverse Solution



Kinematic Singularities

The end-effector mobility locally decreases

Singularities

$$S(q) = \det[J(q)] = S_1(q).S_2(q)\cdots S_{n_s}(q)$$

Singular direction

$S_i = 0$; ζ_i → Infinite effective mass → infinite effective inertia

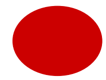
Kinematic Singularities

Singularity Neighborhood

$$S(q) = S_1(q).S_2(q).S_3(q)\cdots S_{n_s}(q)$$

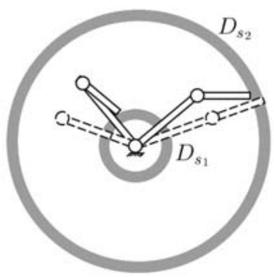
Singularity S_i

$$D_{S_i} = \{ q \mid |S_i(q)| \le S_{0_i} \}$$

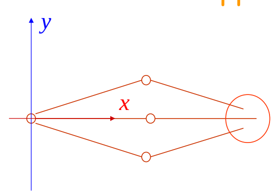


$|S_i| \le S_{0_i}$

Singularity Neighborhood



Approach



In D_S, the robot is treated as redundant w.r.t. motions in the subspace ⊥ to the singular direction

- Along Singular Directions:
  Control in Null Space $\Gamma_{null-space}$
- In subspace ⊥ to singular direction
  Control in sub-O-Space F_{sub-os}

Types of Singularities

Elbow Lock — Wrist Lock — Overhead Lock

Type 1 — Type 2

Types of Singularities

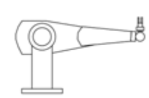
Control Strategy

Type 1

Motion in Null Space

⇒ Motion along/about ζ_i

Control S_i

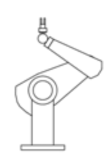


Type 2

Motion in Null Space

⇒ Only changes of ζ_i

Control ζ_i



Singularity Control

$$\Gamma = J_{sub}^T F_{sub} + N_{sub}^T \Gamma_{s_i}$$

where

$N_{sub} = I - \bar{J}_{sub} J_{sub}$ and $\Gamma_{S_i} = -\nabla V_i(S_i)$

Moving to a singularity

Control $S_i(q)$ to reach $S_i = 0$

Moving out of a singularity

Control $\dot{S}_i$ from zero to the desired Velocity at the singularity boundary