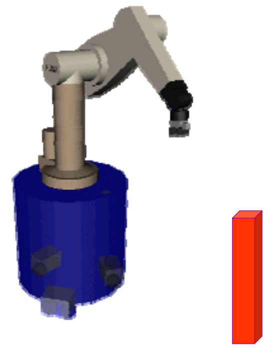
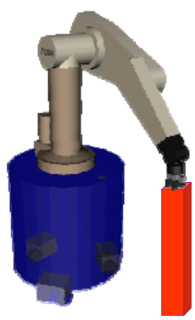
Operational Space Dynamics

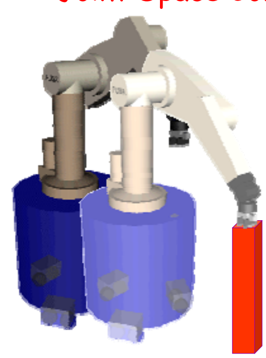
Joint Space Control



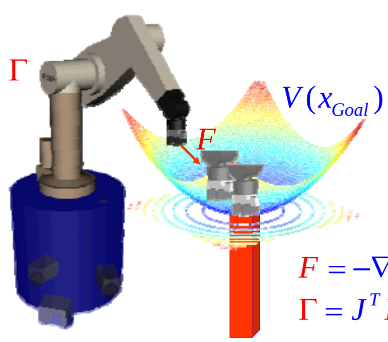
Joint Space Control



Joint Space Control

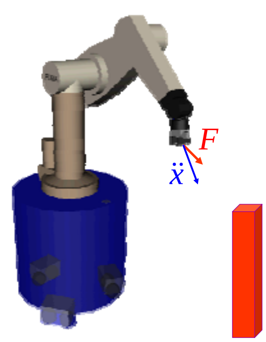


Task-Oriented Control



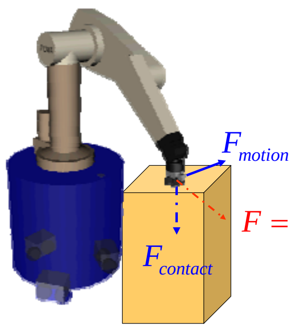
$$F = -\nabla V(x_{Goal})$$
$$\Gamma = J^T F$$

Task-Oriented Dynamics



$$\Lambda \ddot{x} + \mu + p = F$$
$$F = F(\text{dynamics})$$

Unified Motion & Force Control



$$F = F_{motion} + F_{contact}$$

Operational Space Dynamics

$$\Lambda(x)\ddot{x} + \mu(x,\dot{x}) + p(x) = F$$

x: End-Effector Position and Orientation

$\Lambda(x)$: End-Effector Kinetic Energy Matrix

$\mu(x,\dot{x})$: End-Effector Centrifugal and Coriolis forces

$p(x)$: End-Effector Gravity forces

F: End-Effector Generalized forces

Equations of Motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F$$

with $L(x,\dot{x}) = T(x,\dot{x}) - U(x)$

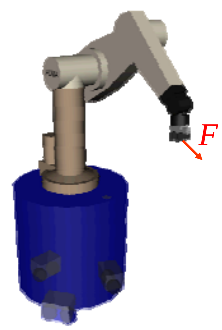
$$\Lambda(x)\ddot{x} + \mu(x,\dot{x}) + p(x) = F$$

μ: Vector of centrifugal and Coriolis Forces

p: Gravity Vector

F: Generalized Operational Forces

End-Effector Control



$$\Gamma = J^T(q)F$$

Passive Systems

$$U_{goal} = \frac{1}{2}k_p\left(x - x_g\right)^T\left(x - x_g\right)$$

System $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial (T - U)}{\partial x} = F$

$$\Downarrow \quad F = -\frac{\partial}{\partial X}\left(U_{goal} - \hat{U}\right)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial \left(T - U_{goal}\right)}{\partial x} = 0$$

Stable

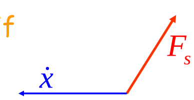
Conservative Forces

Asymptotic Stability

a system $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial \left(T - U_{goal}\right)}{\partial x} = F_s$

is asymptotically stable if

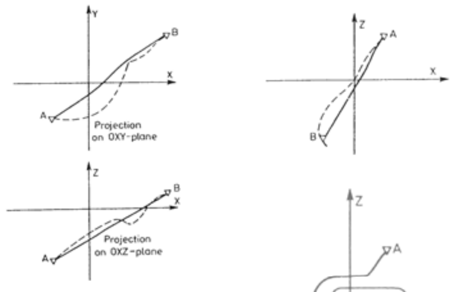
$F_s^T\dot{x} < 0 \quad ; \ for \ \dot{x} \neq 0$

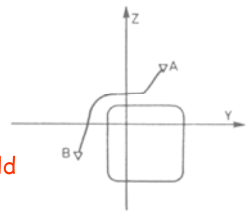


$F_s = -k_v\dot{x} \ \rightarrow \ k_v > 0$

Control

$$F = -k_p\left(x - x_g\right) - k_v\dot{x} + \hat{p}$$

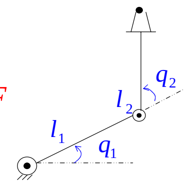




Artificial Potential Field

Example 2-d.o.f arm

$$\Lambda(x)\ddot{x} + \mu(x,\dot{x}) + p(x) = F$$



$$F = -k_p\left(x - x_g\right) - k_v\,\dot{x} + \hat{p}(x)$$

$$\left(m_1^{*}c^2 12 + m_2\right)\ddot{x} + m_1^{*}\ddot{y} + \mu_1 = -k_p\left(x - x_g\right) - k_v\dot{x}$$

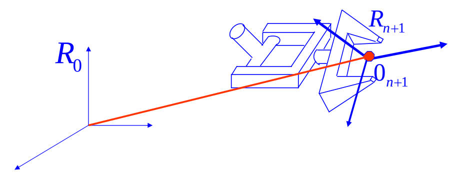
$$\left(m_1^{*}c^2 12 + m_2\right)\ddot{y} + m_1^{*}\ddot{x} + \mu_2 = -k_p\left(y - y_g\right) - k_v\dot{y}$$

Closed loop behavior

$$m_{11}(q)\ddot{x} + k_v\dot{x} + k_p\left(x - x_g\right) = -\left(m_1^{*}\ddot{y} + \mu_1\right)$$

$$m_{22}(q)\ddot{y} + k_v\dot{y} + k_p\left(y - y_g\right) = -\left(m_1^{*}\ddot{x} + \mu_2\right)$$

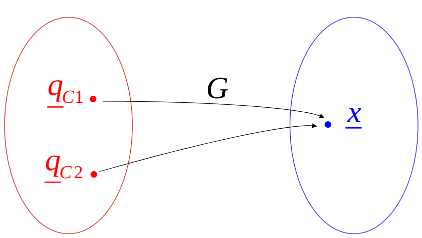
Effector Equations of Motion



Non-Redundant Manipulator ; $n = m_0$

$$x = \left(x_1\, x_2 \ldots x_{m_0}\right)^T$$
$$q = \left(q_1\, q_2 \ldots q_n\right)^T$$
$$x = G\left(q\right)$$

Domain



$$D_q = \prod_{i=1}^{n}\left[\underline{q_i}, \overline{q}_i\right] \qquad D_x = G\left(D_q\right)$$

$$\tilde{D}_x = G\left(\tilde{D}_q\right)$$

$\tilde{D}_q$: Excluding Singularities and such that G is one-to-one

In $\tilde{D}_x$, x_1, x_2, … x_{m_0} form a complete set of configuration parameters for the manipulator.

$x_1, \ldots, x_{m_0}$: system of generalized coordinates

Kinetic Energy

$$T\left(x,\dot{x}\right)=\frac{1}{2}\dot{x}^T\Lambda(x)\,\dot{x}$$

$\Lambda_{m_0\times m_0}(x)$: Kinetic Energy Matrix

$$T_x(x,\dot{x})=\frac{1}{2}\dot{x}^T\Lambda(x)\,\dot{x}$$

Identity

$$T_x(x,\dot{x})\equiv T_q(q,\dot{q})$$

$$\frac{1}{2}\dot{x}^T\Lambda(X)\dot{x}\equiv\frac{1}{2}\dot{q}^T A(q)\,\dot{q}$$

$$\dot{x}=J\dot{q}$$

$$\Lambda(x)=J^{-T}(q)\;A(q)\;J^{-1}(q)$$

$$p(x)=J^{-T}g(q)$$

System $\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{x}}\right)-\frac{\partial\left(T-U\right)}{\partial x}=F$

$$\frac{\partial T}{\partial\dot{x}}=\frac{1}{2}\frac{\partial}{\partial\dot{x}}\left(\dot{x}^T\Lambda\,\dot{x}\right)=\Lambda\,\dot{x}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{x}}\right)=\dot{\Lambda}\dot{x}+\Lambda\ddot{x}$$

$$\frac{\partial T}{\partial x_i}=\frac{1}{2}\dot{x}^T\Lambda_{x_i}\dot{x}$$

$$\mu\left(x,\dot{x}\right)=\dot{\Lambda}\dot{x}-\begin{vmatrix}\frac{1}{2}\dot{x}^T\Lambda_{x_1}\dot{x}\\ \vdots\\ \frac{1}{2}\dot{x}\Lambda_{x_{m_0}}\dot{x}\end{vmatrix}$$

$$\mu(x,\dot{x})=\dot{\Lambda}\dot{x}-m(x,\dot{x})$$

$$m_i=\frac{1}{2}\dot{x}^T\Lambda_{x_i}\dot{x}\qquad;\quad\Lambda=J^{-T}A\,J^{-1}$$

$$\begin{cases}\dot{\Lambda}\dot{x}=J^{-T}\dot{A}\dot{q}-\Lambda h(q,\dot{q})+\dot{J}^{-T}A\dot{q}\\ m(x,\dot{x})=J^{-T}l(q,\dot{q})+\dot{J}^{-T}A\dot{q}\end{cases}$$

where $h\triangleq\dot{J}\dot{q}$

$$l_i\triangleq\frac{1}{2}\dot{q}^T A_{q_i}\dot{q}$$

$$m_i(x,\dot{x})=\frac{1}{2}\dot{x}^T\Lambda_{x_i}\dot{x}$$

$$m(x,\dot{x})=\frac{1}{2}|\,\dot{x}^T J^{-T}A_{x_i}\,J^{-1}\dot{x}\,|+2\,\frac{1}{2}|\,\dot{x}^T J_{x_i}^{-T}A\,J^{-1}\dot{x}\,|$$

$$m(x,\dot{x})=\frac{1}{2}|\,\dot{q}^T A_{x_i}\,\dot{q}\,|+\dot{J}^{-T}A\,\dot{q}$$

$$\dot{q}^T A_{x_i}\dot{q}=\left(\frac{\partial q_1}{\partial x_i}\frac{\partial q_2}{\partial x_i}\cdots\frac{\partial q_n}{\partial x_i}\right).\,|\,\dot{q}^T A_{q_i}\,\dot{q}\,|$$

$$\frac{1}{2}|\,\dot{q}^T A_{x_i}\,\dot{q}\,|=J^{-T}l(q,\dot{q})$$

$$m(x,\dot{x})=J^{-T}l(q,\dot{q})+\dot{J}^{-T}A\,\dot{q}$$

$$\mu = J^{-T}\left(\dot{A}\dot{q} - l(q,\dot{q})\right) - \Lambda h(q,\dot{q})$$

where $h \triangleq \dot{J}\dot{q}$

$$l_i \triangleq \frac{1}{2}\dot{q}^T A_{q_i} \dot{q}$$

$$\mu = J^{-T}(q)\, b(q,\dot{q}) - \Lambda h(q,\dot{q})$$

Joint Space/Operational Space Relationships

$$T_x(x,\dot{x}) \equiv T_q(q,\dot{q})$$

$$\frac{1}{2}\dot{x}^T \Lambda(X)\dot{x} = \frac{1}{2}\dot{q}^T A(q)\dot{q}$$

Using $\dot{x} = J(q)\dot{q}$

$$\frac{1}{2}\dot{q}^T J^T \Lambda\, J\, \dot{q} = \frac{1}{2}\dot{q}^T A\, \dot{q}$$

Joint Space/Operational Space Relationships

$$\Lambda(x) = J^{-T}(q)\, A(q)\, J^{-1}(q)$$

$$\mu(x,\dot{x}) = J^{-T}(q)\, b(q,\dot{q}) - \Lambda(q)\, h(q,\dot{q})$$

$$p(x) = J^{-T}(q)\, g(q)$$

where $h(q,\dot{q}) \doteq \dot{J}(q)\dot{q}$

Λ, μ, and P are all expressed in terms of joint coordinates

The domain $\tilde{D}_x$ can be extended to

$$\bar{D}_x = G\left(\bar{D}_q\right)$$

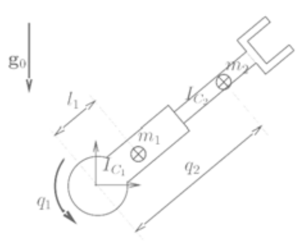
$\bar{D}_q$: domain D_q excluding singularities

Example

$q_2 = d_2$

$$x = \begin{bmatrix} d_2 c1 \\ d_2 s1 \end{bmatrix}$$

$${}^0J = \begin{bmatrix} -d_2 s1 & c1 \\ d_2 c1 & s1 \end{bmatrix}$$



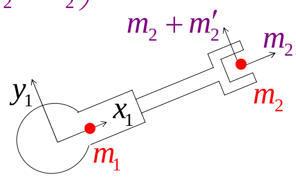
$${}^0J = \begin{bmatrix} -d_2 s1 & c1 \\ d_2 c1 & s1 \end{bmatrix}$$

$${}^0J = \begin{pmatrix} c1 & -s1 \\ s1 & c1 \end{pmatrix} \overbrace{\begin{pmatrix} 0 & 1 \\ d_2 & 0 \end{pmatrix}}^{{}^1J}$$

$${}^1J^{-1} = \begin{pmatrix} 0 & 1/d_2 \\ 1 & 0 \end{pmatrix};$$

$${}^1\Lambda = \begin{pmatrix} 0 & 1 \\ 1/d_2 & 0 \end{pmatrix} \begin{pmatrix} m_{11} & 0 \\ 0 & m_{22} \end{pmatrix} \begin{pmatrix} 0 & 1/d_2 \\ 1 & 0 \end{pmatrix}$$

$$ {}^{1}\Lambda = \begin{pmatrix} m_2 & 0 \\ 0 & m_2 + m_2' \end{pmatrix} $$

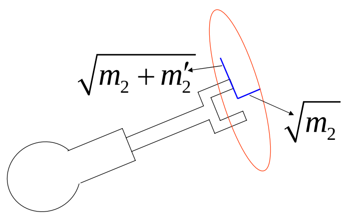


$$ m_2' = \frac{I_{221} + I_{222} + m_1 l_1^2}{d_2^2} $$

$$ {}^{0}\Lambda = \begin{pmatrix} c1 & -s1 \\ s1 & c1 \end{pmatrix} \begin{pmatrix} m_2 & 0 \\ 0 & m_2^+ \end{pmatrix} \begin{pmatrix} c1 & s1 \\ -s1 & c1 \end{pmatrix} $$

$$ m_2^+ = m_2 + m_2' $$

$$ {}^{0}\Lambda = \begin{pmatrix} m_2 + m_2' s1^2 & -m_2' s\, c1 \\ -m_2' s\, c1 & m_2 + m_2' c1^2 \end{pmatrix} $$



$$ {}^{0}\Lambda = \begin{pmatrix} m_2 + m_2' s1^2 & -m_2' s\, c1 \\ -m_2' s\, c1 & m_2 + m_2' c1^2 \end{pmatrix} $$

Nonlinear Dynamic Decoupling

Model

$$ \Lambda(x)\ddot{x} + \mu(x,\dot{x}) + p(x) = F $$

Control Structure

$$ F = \hat{\Lambda}(x)F^* + \hat{\mu}(x,\dot{x}) + \hat{p}(x) $$

Decoupled System

$$ I\,\ddot{x} = F^* $$

with $\Gamma = J^T F$

Dynamic Decoupling

$$ \Lambda(x)\ddot{x} + \mu(x,\dot{x}) + p(x) = F $$

$$ F = \hat{\Lambda}F^* + \hat{\mu}(x,\dot{x}) + \hat{p}(x) $$

$$ I_{m_0}\ddot{X} = \underbrace{\left(\Lambda^{-1}\hat{\Lambda}\right)}_{G(x)} F^* + \Lambda^{-1}\underbrace{\left(\hat{\mu} - \mu\right)}_{\tilde{\mu}_{(x,\dot{x})}} + \Lambda^{-1}\underbrace{\left(P - \hat{P}\right)}_{\tilde{P}(x)} $$

$$ I_{m_0}\ddot{x} = G(x)F^* + \varepsilon(x,\dot{x}) + d(t) $$

$$ G(x) = \Lambda^{-1}\hat{\Lambda} \simeq I + \varepsilon_\Lambda $$

$$ \varepsilon(x,\dot{x}) = \Lambda^{-1}\left(\tilde{\mu} + \tilde{P}\right) $$

$d(t)$: unmodeled disturbances

Perfect Estimates

$$I_{m_0}\,\ddot{x} = F^*$$

F^* input of decoupled end-effector

Goal Position Control

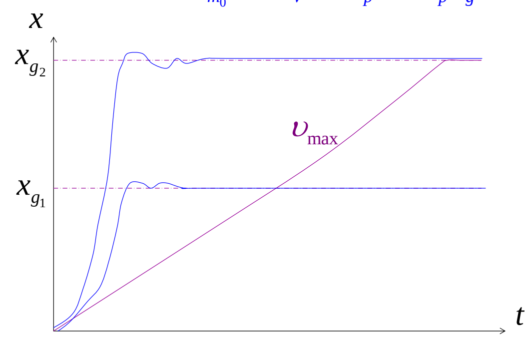
$$F^* = -k_v\,\dot{x} - k_p\left(x - x_g\right)$$

Closed Loop

$$I_{m_0}\,\ddot{x} + k_v\dot{x} + k_p x = k_p x_g$$

Closed Loop

$$I_{m_0}\,\ddot{x} + k_v\dot{x} + k_p x = k_p x_g$$

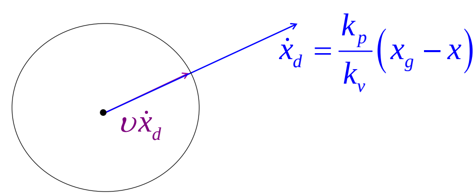


PD Control

$$F^* = -k_v\dot{x} - k_p\left(x - x_g\right)$$

Velocity-Like Control

$$F^* = -k_v\left(\dot{x} - \frac{k_p}{k_v}\left(x_g - x\right)\right)$$

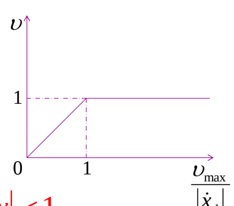


$$F^* = -k_v\left(\dot{x} - \underbrace{\frac{k_p}{k_v}\left(x_g - x\right)}_{\dot{x}_d}\right)$$

$$F^* = -k_v\left(\dot{x} - \upsilon\dot{x}_d\right)$$

with

$$\upsilon = sat\left(\frac{V_{\max}}{\left|\dot{x}_d\right|}\right)$$



$$sat(x) = \begin{cases} x & if\ \left|x\right| < 1 \\ sign(x) & if\ \left|x\right| > 1 \end{cases}$$

Trajectory Tracking

Trajectory: x_d , $\dot{x}_d$, $\ddot{x}_d$

$$F^* = I_{m_0}\ddot{x}_d - k_v(\dot{x} - \dot{x}_d) - k_p(x - x_d)$$

$$(\ddot{x} - \ddot{x}_d) + k_v(\dot{x} - \dot{x}_d) + k_p(x - x_d)$$

or $\ddot{\varepsilon}_X + k_v\dot{\varepsilon}_X + k_p\varepsilon_X = 0$

with $\varepsilon_X = x - x_d$

In joint space

$$\ddot{\varepsilon}_q + k_v \dot{\varepsilon}_q + k_p \varepsilon_q = 0$$

with $\varepsilon_q = q - q_d$