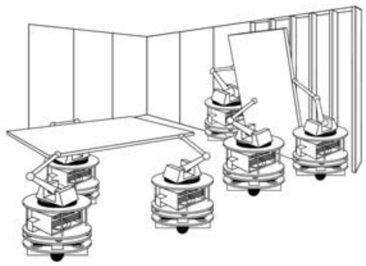
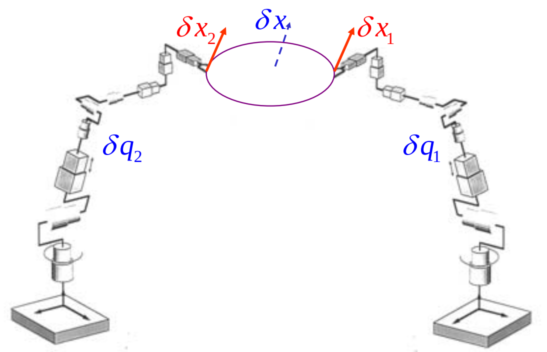


Robot Cooperation

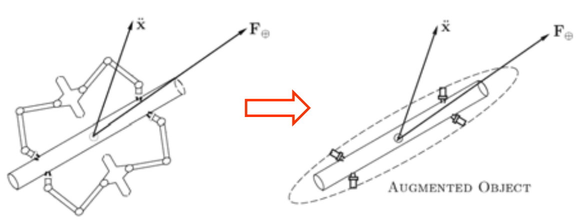
Cooperative Manipulation

- Dynamics: *Augmented Object*
- Internal Forces: *Virtual Linkage*
- Centralized Control: *Fixed-Base Manipulation*
- Decentralized Control: *Mobile-Base Manipulation*

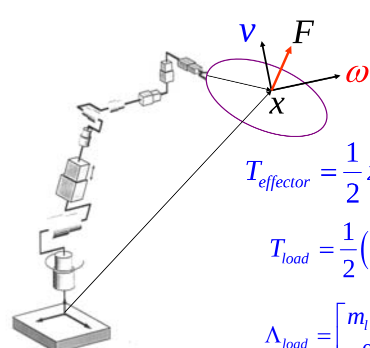
Multi-robot Control Leader/Follower



Augmented Object Model



Effect of a Load

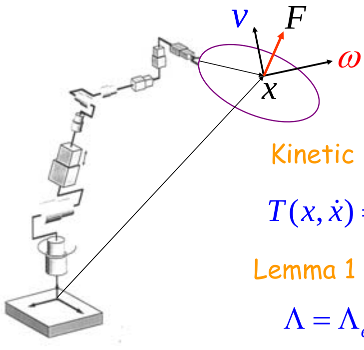


$$T_{effector} = \frac{1}{2}\dot{x}^T \Lambda_{effector}\ \dot{x}$$

$$T_{load} = \frac{1}{2}\left(m_l v^T v + \omega^T I_l \omega\right)$$

$$\Lambda_{load} = \begin{bmatrix} m_l I_3 & 0 \\ 0 & I_l \end{bmatrix}$$

Effect of a Load



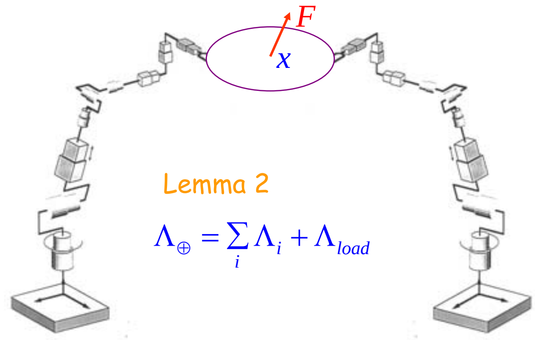
Kinetic Energy

$$T(x,\dot{x}) = T_{effector} + T_{load}$$

Lemma 1

$$\Lambda = \Lambda_{effector} + \Lambda_{load}$$

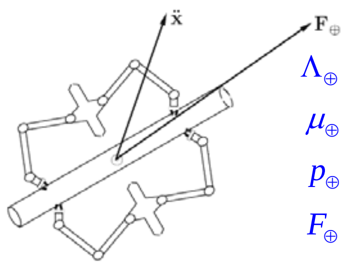
Multi-Arm Control



Lemma 2

$$\Lambda_{\oplus} = \sum_i \Lambda_i + \Lambda_{load}$$

Theorem (Augmented Object)



$$\Lambda_{\oplus} = \sum_i \Lambda_i + \Lambda_{load}$$

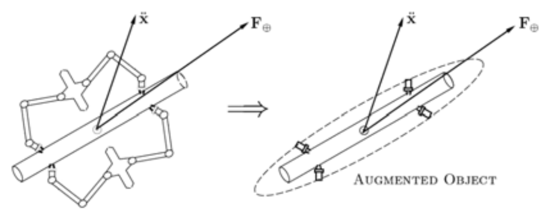
$$\mu_{\oplus} = \sum_i \mu_i + \mu_{load}$$

$$p_{\oplus} = \sum_i p_i + p_{load}$$

$$F_{\oplus} = \sum_i F_i$$

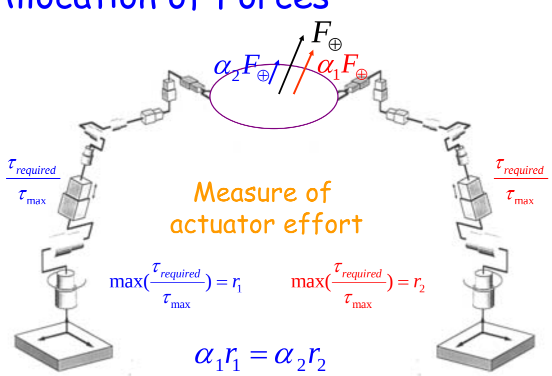
$$\Lambda_{\oplus}\ddot{x} + \mu_{\oplus}(x,\dot{x}) + p_{\oplus}(x) = F_{\oplus}$$

Augmented Object Model



$$\Lambda_{\oplus}\ddot{x} + \mu_{\oplus}(x,\dot{x}) + p_{\oplus}(x) = F_{\oplus}$$

Allocation of Forces



Measure of actuator effort

$$\max\left(\frac{\tau_{required}}{\tau_{\max}}\right) = r_1 \qquad \max\left(\frac{\tau_{required}}{\tau_{\max}}\right) = r_2$$

$$\alpha_1 r_1 = \alpha_2 r_2$$

Allocation of Forces

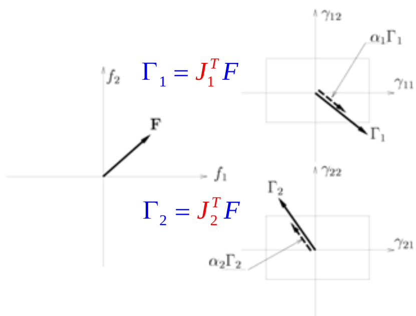
For N robots

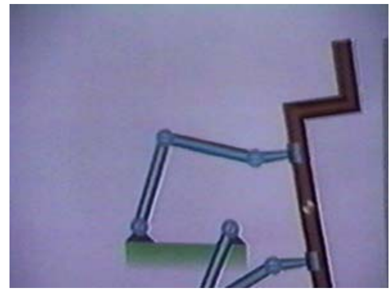
$$\alpha_1 r_1 = \alpha_2 r_2 = \cdots = \alpha_N r_N$$

$\Rightarrow \alpha_i$: Minimized overall effort

$$\alpha_i = \frac{\beta_i}{\beta_1 + \beta_2 + \cdots + \beta_N}; \text{ where } \beta_i = \frac{r_1 r_2 \cdots r_N}{r_i}$$

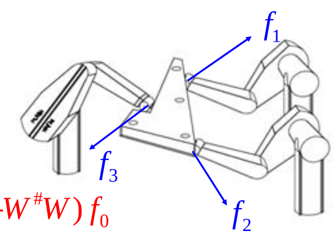
Allocation of Forces





Internal Forces

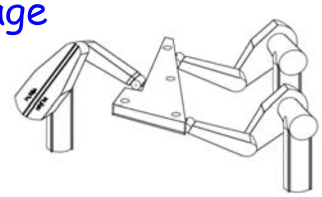
$$f_r = W f$$



$$f = W^{\#} f_r + (I - W^{\#} W) f_0$$

$$f_{\text{internal}} = (I - W^{\#} W) f$$

Virtual Linkage

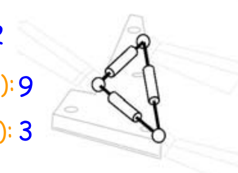


Actuator DOF: 18

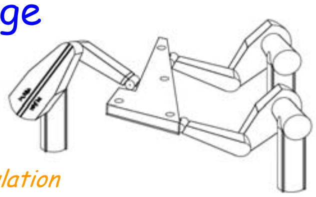
Resultant Force: 6

Actuator Redundancy: 12

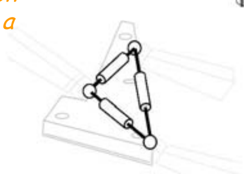
- Internal Moments (3N): 9
- Internal Forces (3N-6): 3



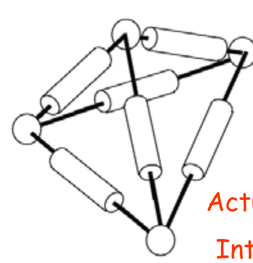
Virtual Linkage



For an N-grasp manipulation task, the virtual linkage is a 6(N-1) DOF mechanism, whose actuated joints characterize the internal forces and moments.



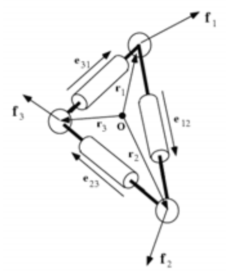
A Four-Grasp Virtual Linkage



Actuation: 24DOF

Internal: 18DOF

Virtual Linkage Resultant Forces



$$\begin{bmatrix} f_r \\ m_r \end{bmatrix} = W_{(6\times18)} \begin{bmatrix} f \\ m \end{bmatrix}$$

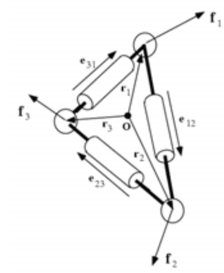
$$W_{(6\times18)} = \begin{bmatrix} W_{f\,(6\times9)} & W_{m(6\times9)} \end{bmatrix}$$

$$W_f = \begin{bmatrix} I_3 & I_3 & I_3 \\ \hat{r}_1 & \hat{r}_2 & \hat{r}_3 \end{bmatrix}$$

$$W_m = \begin{bmatrix} 0 & 0 & 0 \\ I_3 & I_3 & I_3 \end{bmatrix}$$

Virtual Linkage Internal Forces

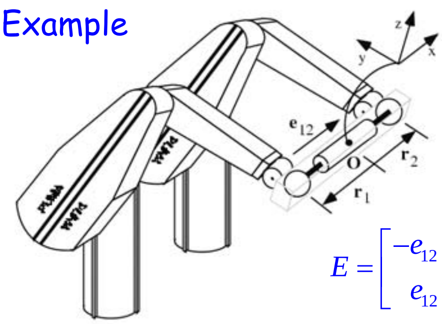
$$f = E_{(9\times3)}\, t$$



$$E = \begin{bmatrix} -e_{12} & 0 & e_{31} \\ e_{12} & -e_{23} & 0 \\ 0 & e_{23} & -e_{31} \end{bmatrix}$$

$$t = \bar{E}_{(3\times9)}\, f$$

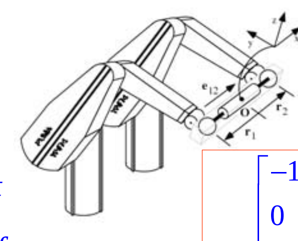
Example



$$E = \begin{bmatrix} -e_{12} \\ e_{12} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{E} = \frac{1}{2}\begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$



$$f = E\ t$$

$$t = \bar{E}\ f$$

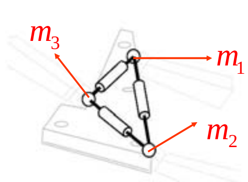
$$\bar{E} = \frac{1}{2}\begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$t = 1$$

$$f = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Virtual Linkage Internal Moments

$$\tau = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}$$



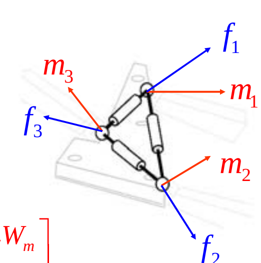
Grasp Description Matrix

$$\begin{bmatrix} f_r \\ m_r \\ t \\ \tau \end{bmatrix} = G_{(18\times18)} \begin{bmatrix} f \\ m \end{bmatrix}$$

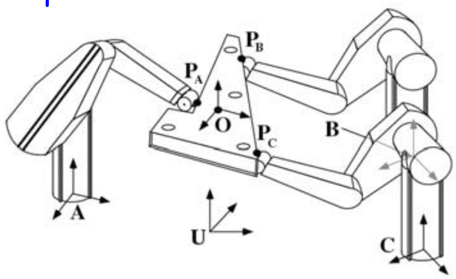
$$G = \begin{bmatrix} W_f & W_m \\ \bar{E} & 0_{3\times9} \\ 0_9 & I_9 \end{bmatrix}$$

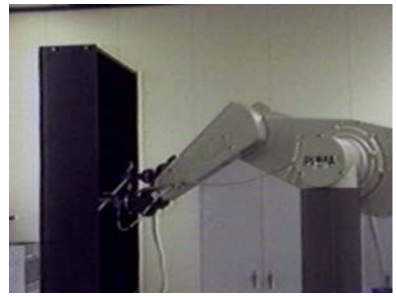
full rank

$$G^{-1} = \begin{bmatrix} \bar{W}_f & E & -\bar{W}_f W_m \\ 0_{9\times6} & 0_{9\times3} & I_9 \end{bmatrix}$$

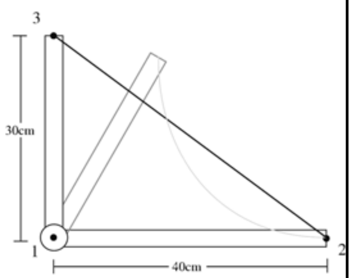


Example





Example



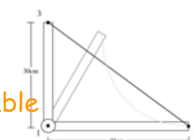
$$e_{12} = [1\ \ 0\ \ 0]^T$$

$$e_{23} = [-0.8\ \ 0.6\ \ 0]^T$$

$$e_{31} = [0\ \ -1\ \ 0]^T$$

$$r_1 = [0\ \ 0\ \ 0]^T \quad r_2 = [0.4\ \ 0\ \ 0]^T \quad r_3 = [0\ \ 0.3\ \ 0]^T$$

Requirements



- 10 Newtons tension in cable
- 5 Newtons compression in horizontal member
- *no tension in vertical member*
- *zero internal moments*
- *no resultant forces or moments*

$$f = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -5.00 \\ 10.00 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix}$$

$$W_f = \begin{vmatrix} 1.00 & 0 & 0 & 1.00 & 0 & 0 & 1.00 & 0 & 0 \\ 0 & 1.00 & 0 & 0 & 1.00 & 0 & 0 & 1.00 & 0 \\ 0 & 0 & 1.00 & 0 & 0 & 1.00 & 0 & 0 & 1.00 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.30 \\ 0 & 0 & 0 & 0 & 0 & -0.40 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.40 & 0 & -0.30 & 0 & 0 \end{vmatrix}$$

$$\overline{W}_f = \begin{vmatrix} 0.39 & -0.08 & 0 & 0 & 0 & 0.60 \\ -0.08 & 0.44 & 0 & 0 & 0 & -0.80 \\ 0 & 0 & 1.00 & -3.33 & 2.50 & 0 \\ 0.39 & -0.08 & 0 & 0 & 0 & 0.60 \\ 0.16 & 0.12 & 0 & 0 & 0 & 1.60 \\ 0 & 0 & 0.00 & 0 & -2.50 & 0 \\ 0.21 & 0.16 & 0 & 0 & 0 & -1.20 \\ -0.08 & 0.44 & 0 & 0 & 0 & -0.80 \\ 0 & 0 & 0 & 3.33 & 0 & 0 \end{vmatrix}$$

$$W_m = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.00 & 0 & 0 & 1.00 & 0 & 0 & 1.00 & 0 & 0 \\ 0 & 1.00 & 0 & 0 & 1.00 & 0 & 0 & 1.00 & 0 \\ 0 & 0 & 1.00 & 0 & 0 & 1.00 & 0 & 0 & 1.00 \end{vmatrix}$$

$$E = \begin{vmatrix} -1.00 & 0 & 0 \\ 0 & 0 & -1.00 \\ 0 & 0 & 0 \\ 1.00 & 0.80 & 0 \\ 0 & -0.60 & 0 \\ 0 & 0 & 0 \\ 0 & -0.80 & 0 \\ 0 & 0.60 & 1.00 \\ 0 & 0 & 0 \end{vmatrix}$$

$$\bar{E} = \begin{vmatrix} -0.61 & -0.08 & 0 & 0.39 & 0.16 & 0 & 0.21 & -0.08 & 0 \\ 0.27 & 0.20 & 0 & 0.27 & -0.40 & 0 & -0.53 & 0.20 & 0 \\ -0.08 & -0.56 & 0 & -0.08 & 0.12 & 0 & 0.16 & 0.44 & 0 \end{vmatrix}$$

$$G = \begin{vmatrix}
1.00 & 0 & 0 & 1.00 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1.00 & 0 & 0 & 1.00 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1.00 & 0 & 0 & 1.00 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.30 & 1.00 & 0 & 0 & 1.00 & 0 & 0 & 1.00 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.40 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & 1.00 & 0 & 0 & 1.00 & 0 \\
0 & 0 & 0 & 0 & 0.40 & 0 & -0.30 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & 1.00 & 0 & 0 & 1.00 \\
-0.61 & -0.08 & 0 & 0.39 & 0.16 & 0 & 0.21 & -0.08 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.27 & 0.20 & 0 & 0.27 & -0.40 & 0 & -0.53 & 0.20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.08 & -0.56 & 0 & -0.08 & 0.12 & 0 & 0.16 & 0.44 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00
\end{vmatrix}$$

$$G^{-1} = \begin{vmatrix}
0.39 & -0.08 & 0 & 0 & 0 & 0.60 & -1.00 & 0 & 0 & 0 & 0 & 0.60 & 0 & 0 & 0.60 & 0 & 0 & 0.60 \\
-0.08 & 0.44 & 0 & 0 & 0 & -0.80 & 0 & 0 & -1.00 & 0 & 0 & -0.80 & 0 & 0 & -0.80 & 0 & 0 & -0.80 \\
0 & 0 & 1.00 & -3.33 & 2.50 & 0 & 0 & 0 & 0 & -3.33 & 2.50 & 0 & -3.33 & 2.50 & 0 & -3.33 & 2.50 & 0 \\
0.39 & -0.08 & 0 & 0 & 0 & 0.60 & 1.00 & 0.80 & 0 & 0 & 0 & 0.60 & 0 & 0 & 0.60 & 0 & 0 & 0.60 \\
0.16 & 0.12 & 0 & 0 & 0 & 1.60 & 0 & -0.60 & 0 & 0 & 0 & 1.60 & 0 & 0 & 1.60 & 0 & 0 & 1.60 \\
0 & 0 & 0.00 & 0 & -2.50 & 0 & 0 & 0 & 0 & 0 & -2.50 & 0 & 0 & -2.50 & 0 & 0 & -2.50 & 0 \\
0.21 & 0.16 & 0 & 0 & 0 & -1.20 & 0 & -0.80 & 0 & 0 & 0 & -1.20 & 0 & 0 & -1.20 & 0 & 0 & -1.20 \\
-0.08 & 0.44 & 0 & 0 & 0 & -0.80 & 0 & 0.60 & 1.00 & 0 & 0 & -0.80 & 0 & 0 & -0.80 & 0 & 0 & -0.80 \\
0 & 0 & 0 & 3.33 & 0 & 0 & 0 & 0 & 0 & 3.33 & 0 & 0 & 3.33 & 0 & 0 & 3.33 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 \\
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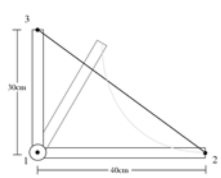
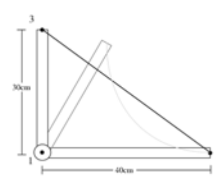
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00
\end{vmatrix}$$

$$f = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -5.00 \\ 10.00 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} \qquad G^{-1}\ f = \begin{vmatrix} 5.00 \\ 0 \\ 0 \\ 3.00 \\ -6.00 \\ 0 \\ -8.00 \\ 6.00 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix}$$



$$f_{applied} = \begin{vmatrix} -3.00 \\ -3.00 \\ 0 \\ 3.00 \\ -3.00 \\ 0 \\ -3.00 \\ 3.00 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} \qquad G\ f_{applied} = \begin{vmatrix} -3.00 \\ -3.00 \\ 0 \\ 0 \\ 0 \\ -0.30 \\ 1.88 \\ 2.80 \\ 2.16 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix}$$



Augmented Object

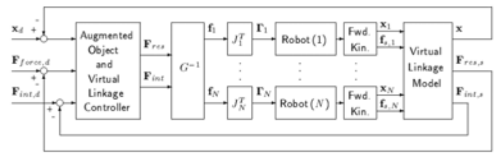
$$\Lambda_{\oplus}\ddot{x} + \mu_{\oplus}(x,\dot{x}) + p_{\oplus}(x) = F_{\oplus}$$

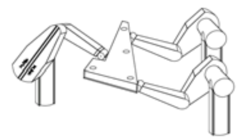
Virtual Linkage: Grasp Matrix

$$\begin{bmatrix} f_{res} \\ f_{int} \end{bmatrix} = G_{(6N\times 6N)} \begin{bmatrix} f_1 \\ \vdots \\ f_N \end{bmatrix}$$

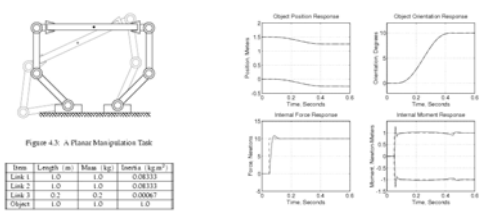
$$\begin{bmatrix} f_1 \\ \vdots \\ f_N \end{bmatrix} = G^{-1}_{(6N\times 6N)} \begin{bmatrix} f_{res} \\ f_{int} \end{bmatrix}$$

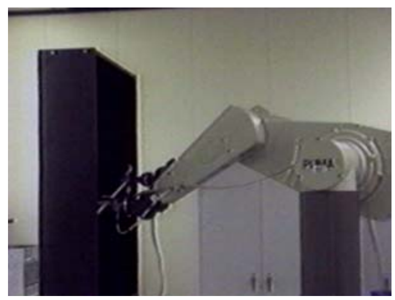
Centralized Control Structure



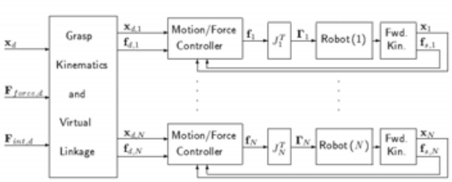


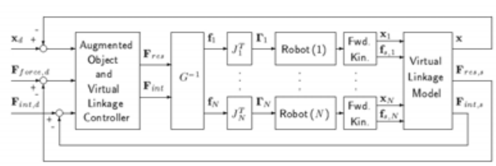
Example

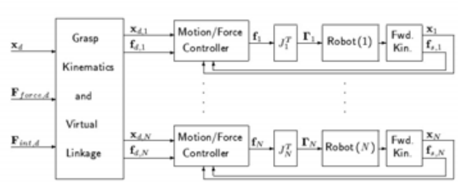


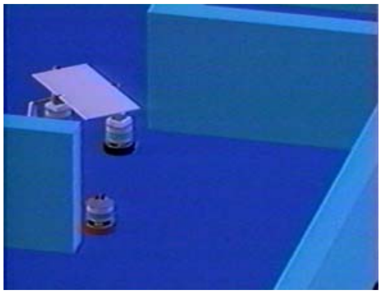


Decentralized Control Structure

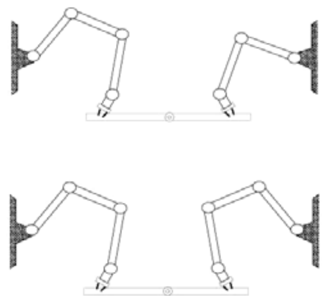








Redundancy



Grubler formula

$$n_S = n_0\left(n_{link} - 1\right) - \left(n_0 - 1\right) n_{joint}$$

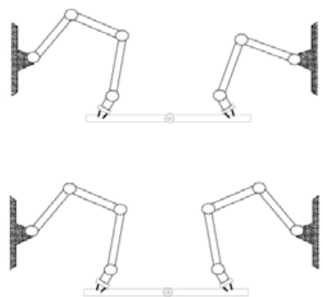
with

$n_0 = 3$ planar

$n_0 = 6$ spatial

Redundancy

$$n_{system} = 3\left(n_{link} - 1\right) - 2n_{joint}$$



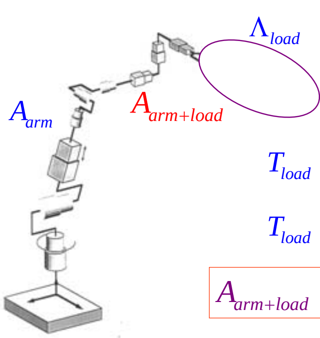
$n_{system} = 4$

$n_{object} = 3$

$n_{system} = 5$

$n_{object} = 3$

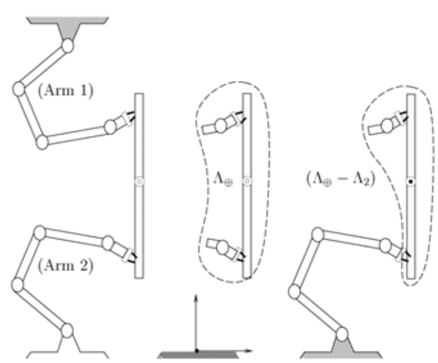
Reflected Load



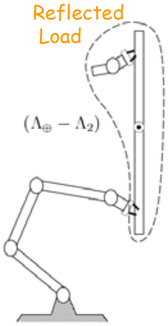
$$T_{load} = \frac{1}{2}\dot{x}^T \Lambda_{load}\, \dot{x}$$

$$T_{load} = \frac{1}{2}\dot{q}^T (J^T \Lambda_{load}\, J)\dot{q}$$

$$A_{arm+load} = A_{arm} + J^T \Lambda_{load} J$$



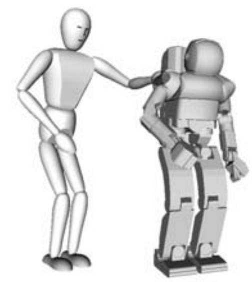
Multiple Redundant Robots



$$A_{+_i} = A_i + J_i^T \left(\Lambda_\oplus - \Lambda_i\right) J_i$$

$$\bar{J}_i = A_{+_i}^{-1} J_i^T \left(J_i A_{+_i}^{-1} J_i^T\right)^{-1}$$

$$\Gamma_i = \alpha J_i^T F + \left(I - J_i^T \bar{J}_i^T\right) \Gamma_{i0}$$



Working with the Human