Array Programming

CS242

Lecture 15

Review

- We’ve studied two function-based programming calculi
  - SKI combinators
  - Lambda Calculus

- In practice, lambda calculus has proven far more popular
  - The basis for functional languages
  - Used to model and understand most programming features
    - State, exceptions, continuations, …

- But combinator programming is not just theoretical

Overview

- In practice, combinator programming is used most with collections
  - And particularly arrays

- Benefits
  - Conciseness: Bulk operations over the entire collection
    - Iteration/recursion is “baked in” to the operations
  - Performance: Leave the details of the implementation the underlying system
    - Might be very different for different hardware, e.g., CPUs or GPUs

An Example

- Two combinators
  - o   function composition
  - map   apply a function to every element of a list/array

- Semantics
  - map f [1, 2, 3] = [f 1, f 2, f 3]
  - map (+ 1) [1, 2, 3] = [2, 3, 4]

Consider the program:

(map f) o (map g)

In a conventional language

```
array a[n],b[n],c[n]
for i = 1,a.len {
    b[i] = f(a[i])
}
for j = 1,a.len {
    c[j] = g(b[j])
}
```

Comparison, Part I

Consider the program:

```
(map f) o (map g)
```

Much more concise!

Why: Conventional version uses general control structures. Combinator version uses a higher-order function (map) that captures exactly the specific iteration pattern needed.

In a conventional language

```
array a[n],b[n],c[n]
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}
for j = 1,a.len {
    c[j] = g(b[j])
}
```

Comparison, Part I

Consider the program:

(map f) o (map g)

Easier to optimize!

An algebraic law:

(map f) o (map g) = map (f o g)

This transformation eliminates the intermediate list/array.

Much harder to recognize when written with explicit for-loops.

In a conventional language

```
array a[n],b[n],c[n]
for i = 1,a.len {
    b[i] = f(a[i])
}
for j = 1,a.len {
    c[j] = g(b[j])
}
```

A Digression

An algebraic law:

(map f) o (map g) = map (f o g)

But what if we are programming in some monad?

E.g., with state or exceptions?

History (Review)

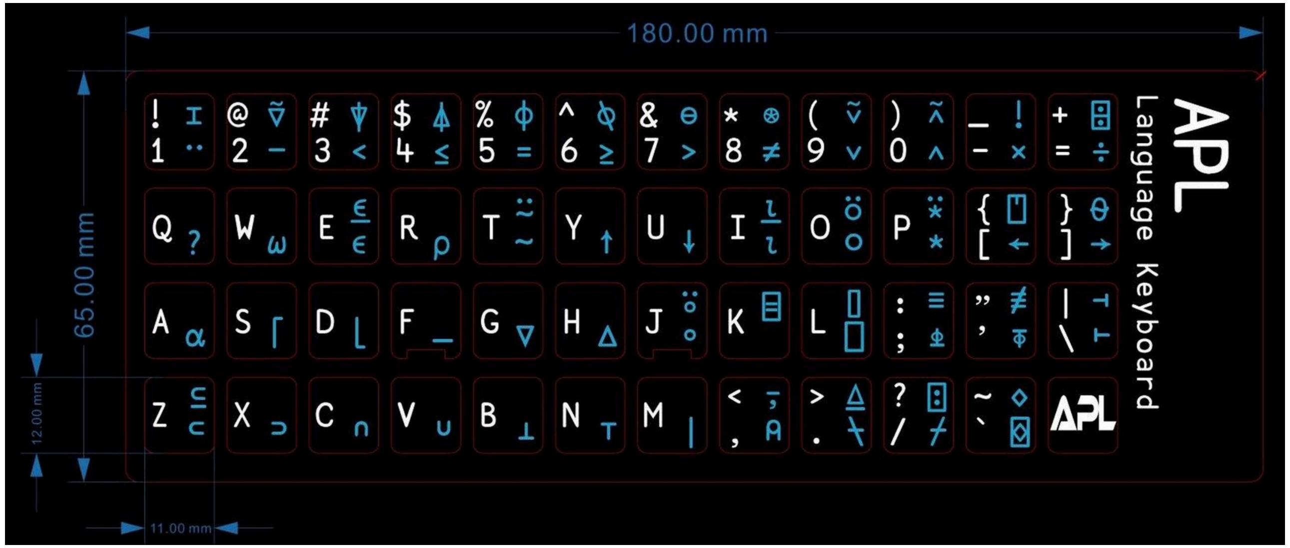
- First combinator-based programming language was APL
  - "A Programming Language"
  - Designed by Ken Iverson in the 1960's

- Designed for expressing pipelines of operations on bulk data
  - Array programming
  - Basic data type is the multidimensional array

- The average of a vector of numbers: `{(+/ω)÷≢ω}`

APL's Legacy

- Marketed by IBM starting in 1968
  - Eventually other companies also offered APL products
- Very influential
  - At least 50 subsequent array programming langauages
  - Recent increased interest with the rising importance of array-based applications (e.g., deep learning) and GPUs
- Trivia: You can buy special APL keyboards today!



From APL to NumPy

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  - And particularly arrays

- Benefits
  - Conciseness: Bulk operations over the entire collection
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- The most popular of these interfaces today is NumPy
  - But note, python has imperative features
  - So programs tend to be a mix of styles, including using variables, state, etc.

A Brief NumPy Tutorial

A short overview of NumPy arrays

- Defining
- Shape
- Broadcasting
- Views
- Filters

Using NumPy

```
# This line will always appear in a NumPy program
import numpy as np
```

Defining an Array

```
import numpy as np

# initialize an array A of 10 elements with the integers 0..9
A = np.arange(0,10)
```

Example: Adding Arrays

```
import numpy as np
A = np.arange(0,10)

# addition is pointwise if the dimensions match
np.add(A,A)
```

Reshaping

```
import numpy as np
A = np.arange(0,10)

# Reshaping is a general operation that changes array dimensions.
# Normally defines a view: creates an alias of the array -- does
# not make a copy.

# view the elements of A as a 2x5 array
A.reshape(2,5)

# view the elements of A as a 10x1 (column) array
A.reshape(10,1)

# Note that reshaping would be very difficult in a static type system!
```

Example: Outer Product

```
import numpy as np
A = np.arange(0,10)

# We can use a combination of reshape and broadcast to define a
# concise outer product.

np.multiply(A,A.reshape(10,1))
```

Broadcasting

- Broadcasting takes two arrays of possibly different dimensions and casts them to arrays of the same dimension

- Rules for broadcast in an array operation A op B
  - If one array has fewer dimensions, add dimensions of size 1 until both have the same number of dimensions
  - For each dimension i
    - If A and B have the same size in dimension i, do nothing
    - If one of A and B has size 1 in dimension i, replicate data in the dimension to the same size as the other array
    - If A and B have different sizes in dimension i and neither is 1, throw an error

- Example
  - A * 5
  - The 5 (a 0-D array) is promoted to a 1-D array of 5's of the same length as A

Slicing

```
import numpy as np
A = np.arange(0,10)

# slicing defines views (aliases) of subsets of an array
A[3:]     # slice of 4th element to the end of the array
A[:-3]    # slice up to the 4th element from the end of the array
A[1:-1]  # slice of all but the first and last elements of the array
A.reshape(2,5)[:,1:3]     # slicing in multiple dimensions
A.reshape(2,5)[0:2,1:3] # same slice written a different way
```

Example: Moving Average

```
import numpy as np
A = np.arange(0,10)

# cumulative sum is one of many NumPy built-in array functions
B = np.cumsum(A)

# moving average of A with a window of size 3
(B[3:] – B[:-3] )  / 3.0
```

Masks

```
import numpy as np
A = np.arange(0,10)

# Using an array in a predicate returns an array of Boolean results
# Here broadcasting promotes 5 to a 1D array of 5's
A > 5
A <= 5
(2 * A) == (A ** 2)
```

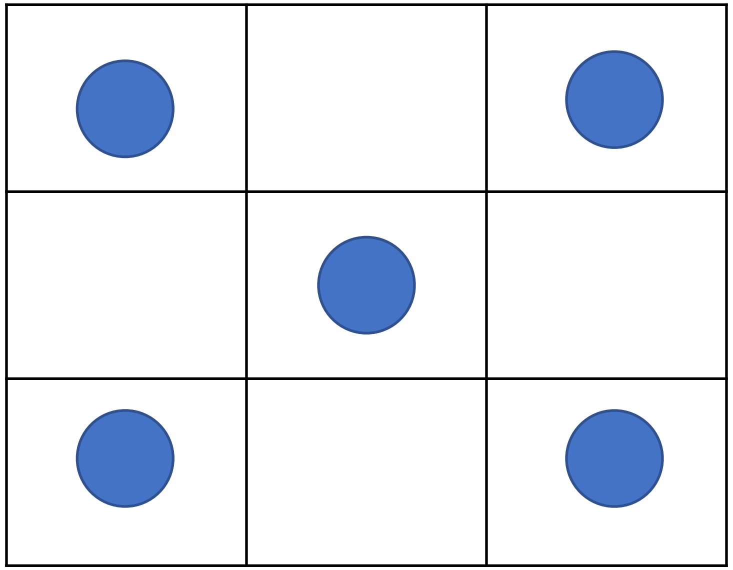
Filters

```
import numpy as np
A = np.arange(0,10)

# Boolean arrays can be used as array indices to filter arrays
A[A > 5]                # elements of A that are > 5
A[A <= 5]               # elements of A that are <= 5
A[(2 * A) == (A ** 2)]  # elements x of A where 2*x == x ** 2
```

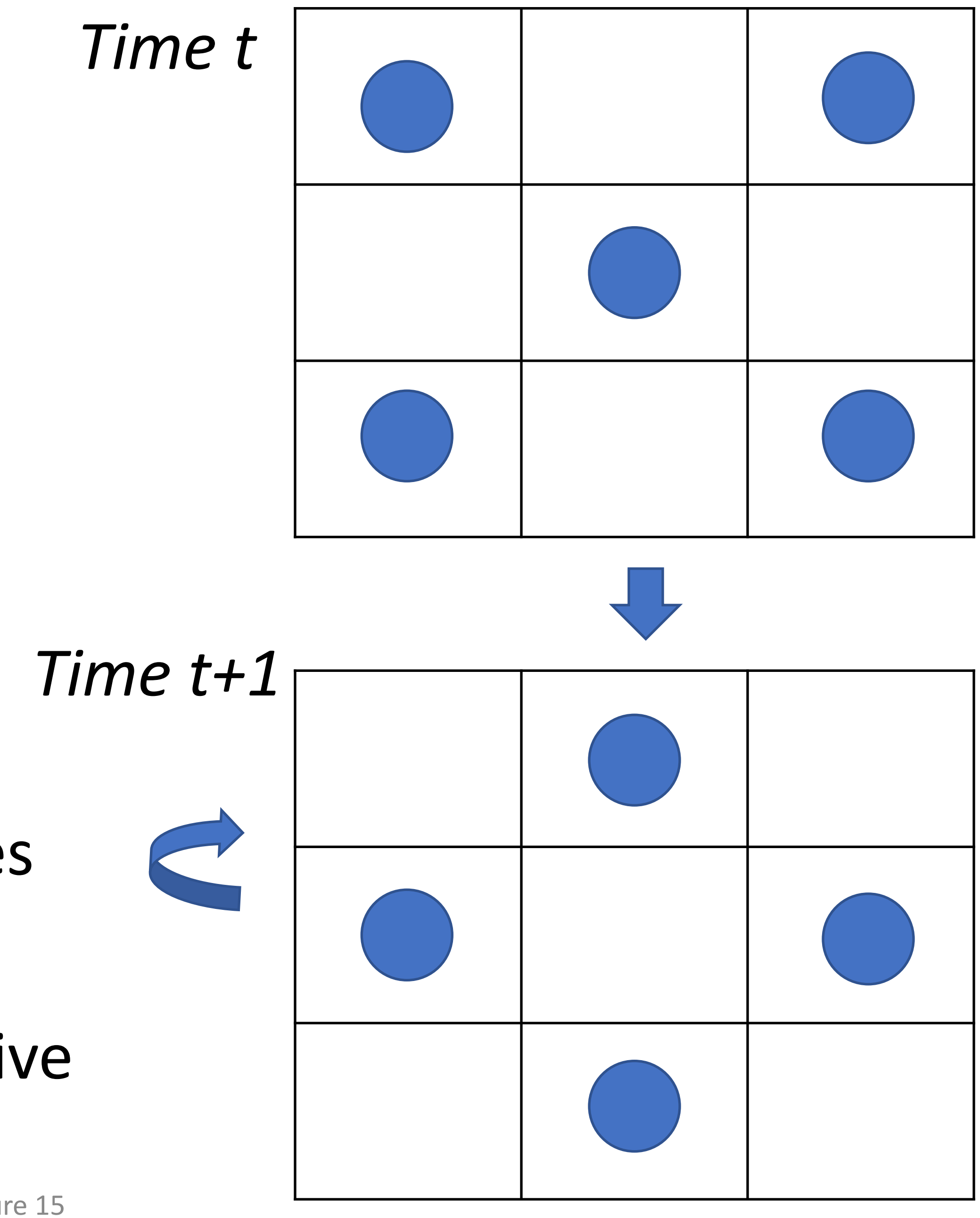
A Bigger Example: The Game of Life

- The Game of Life is played on 2D grid in time steps
- Grid cells are either *live* or *dead*
- A cell is live or dead at time *t+1* based on its neighbors at time *t*
  - Cells at the world's edge are always dead
- Defined by George Conway in 1969
  - An early example of cellular automata



Rules

- A live cell with < 2 neighbors dies
    - From loneliness
- A live cell with > 3 neighbors dies
    - From overcrowding
- A live cell with 2 or 3 neighbors survives
- A dead cell with 3 neighbors becomes live



The Game of Life

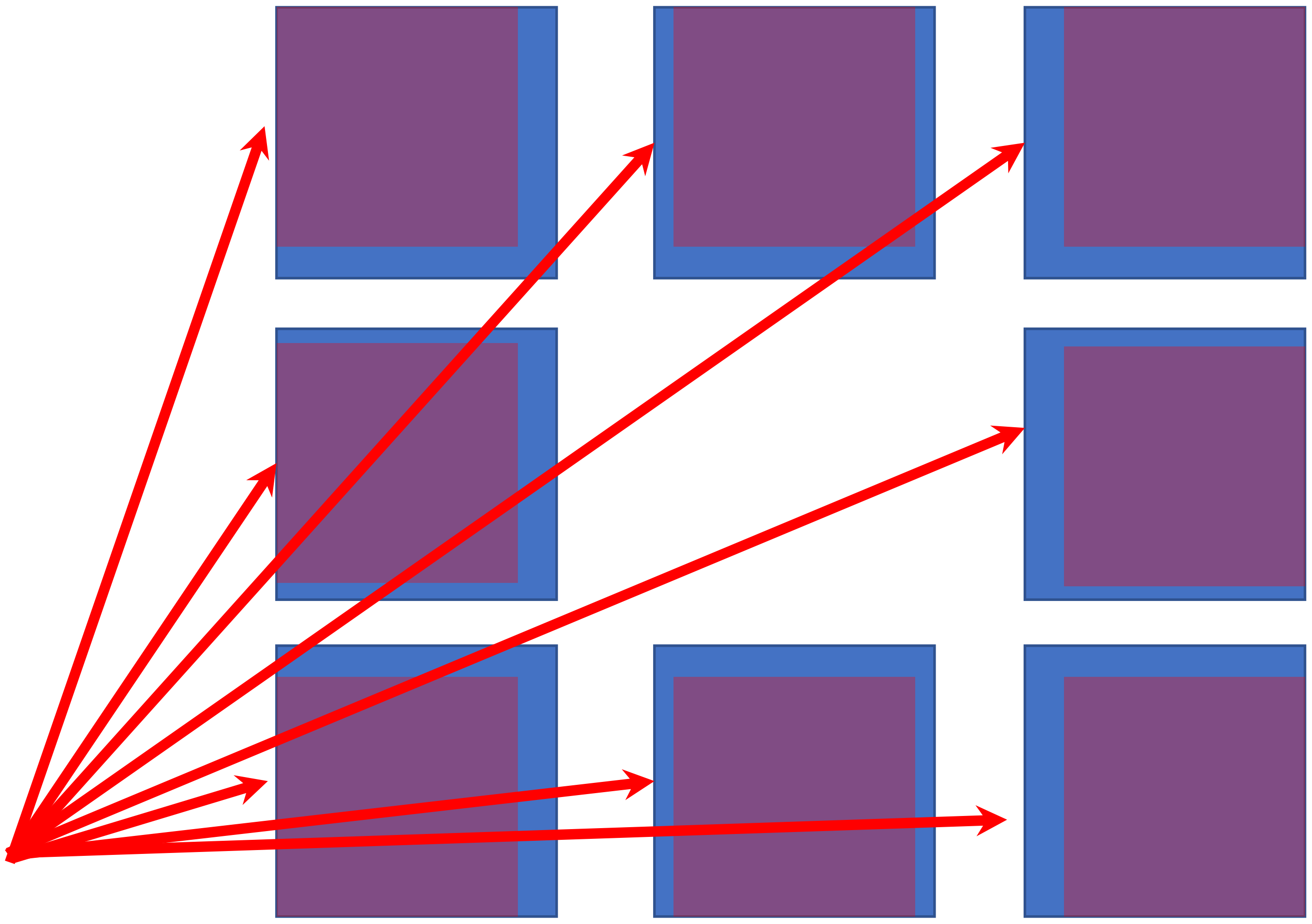
```
import numpy as np
Z = np.zeros((300, 600))
Z[1:-1,1:-1] = np.random.randint(0,2,np.shape(Z[1:-1,1:-1]))      # 0 is dead, 1 is live

while True:
    N = (Z[0:-2, 0:-2] + Z[0:-2, 1:-1] + Z[0:-2, 2:] +
         Z[1:-1, 0:-2]                 + Z[1:-1, 2:] +
         Z[2:  , 0:-2] + Z[2:  , 1:-1] + Z[2:  , 2:])
    birth = (N == 3) & (Z[1:-1, 1:-1] == 0)
    survive = ((N == 2) | (N == 3)) & (Z[1:-1, 1:-1] == 1)
    Z[:,:] = 0
    Z[1:-1, 1:-1][birth | survive] = 1
```

Picture

N = (Z[0:-2, 0:-2] + Z[0:-2, 1:-1] + Z[0:-2, 2:] +
Z[1:-1, 0:-2] + Z[1:-1, 2:] +
Z[2: , 0:-2] + Z[2: , 1:-1] + Z[2: , 2:])

Summing these 8 subarrays computes the number of live neighbors for each cell in the interior of the space.



Explanation

…

```
# N is a 2D array of the number of neighbors of each cell
# birth is a 2D Boolean array; a cell is true if it is has 3 neighbors and is dead
birth = (N == 3) & (Z[1:-1, 1:-1] == 0)

# survive is a 2D Boolean array; a cell is true if it is has 2 or 3 neighbors and is live
survive = ((N == 2) | (N == 3)) & (Z[1:-1, 1:-1] == 1)

# create a new generation
# the interior cells of Z are live if they are born or survive the previous time step
Z[:,:] = 0
Z[1:-1, 1:-1][birth | survive] = 1
```

The Game of Life

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        Z[2:  , 0:-2] + Z[2:  , 1:-1] + Z[2:  , 2:])
   birth = (N == 3) & (Z[1:-1, 1:-1] == 0)
   survive = ((N == 2) | (N == 3)) & (Z[1:-1, 1:-1] == 1)
   Z[:,:] = 0
   Z[1:-1, 1:-1][birth | survive] = 1
```

Summary

- Combinator calculi are important in practice for array/collection programming
  - Where thinking in terms of bulk operations with built-in iteration is useful
  - Often useful in parallel implementations
    - Because the combinators can be high-level enough that the programmer doesn't need to be aware of parallelism at all
- Combinators are also important in program transformations
  - Much easier to design combinator-based transformation systems
  - Some compilers (Haskell's GHC) even translate into an intermediate combinator-based form for some optimizations