The ray density estimation of a CT system by a supervised learning algorithm

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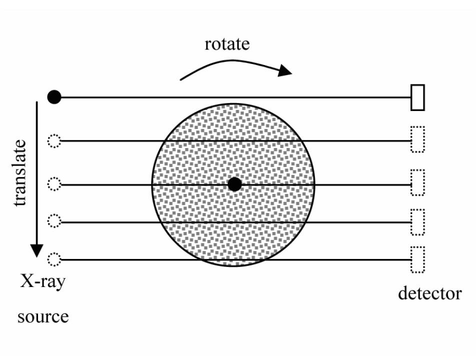
1. Topic

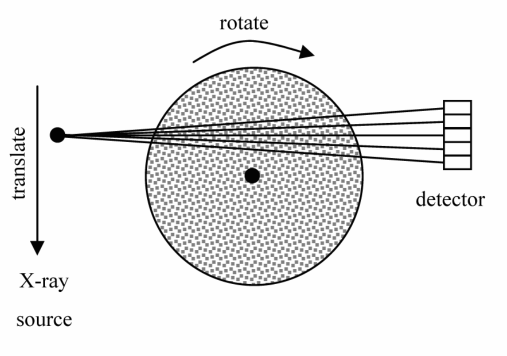
My topic is to find the ray density of a new CT system by using the learning algorithm.

2. Background

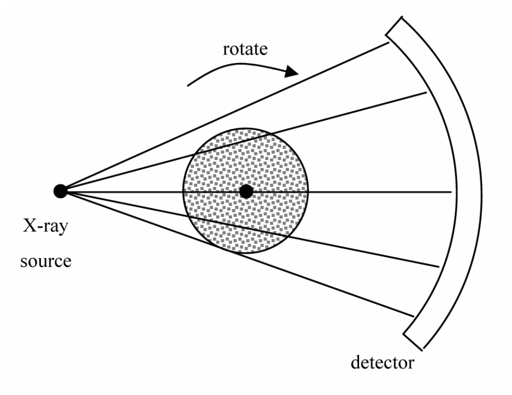
Since the development of the CT scanner, the faster scan time was the highest priority to achieve, and therefore, there have been a lot of efforts to reduce the scan time. The first generation CT scanner (Figure 1(a)) uses one pencil beam, and the projection data were acquired by translating the x-ray source and detector linearly. After the completion of the linear measurements, the x-ray source and detector rotated to the next angular position to acquire the next set of measurements. A faster scan time could be achieved by using multiple pencil beams. Since this second generation CT scanner (Figure 1 (b)) used a small fan beam, the data acquisition time was reduced by the same factor of the increased detector numbers. However, a translation-rotation principle was still employed for data acquisition. The third generation scanner (Figure 1 (c)) used a large number of detectors and a single source. Since the detector size was sufficient to cover the entire object, a translation-rotation principle was not employed. The elimination of the translation step reduced the scan time significantly, and nearly all of the state-of-the-art scanners on the market today are third generation. The potential problem of the third generation system is the high detector cost, and high scatter-to-primary ratio. In order to combat these problems, a different type of CT geometry can be imagined as shown in Figure 1 (d). In this system, instead of using one source and many detector cells, it uses many sources and smaller number of detector cells. For example, if the third generation CT system uses 1000 detector cells, the new system (Figure 1(d)) may employ 50 detector cells, and 20 sources so that the detector cost can be reduced by a factor of 20.



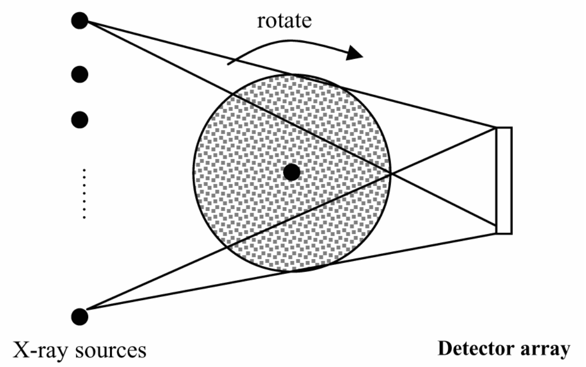
(a) First generation CT



(b) Second generation CT



(c) Third generation CT



(d) New CT system

Figure 1. Different types of CT scanner

3. Supervised learning and reconstruction

The reconstruction algorithm for the third generation CT system was already developed [1]. It uses the filtered backprojection algorithm, and the ray density is compensated before the filtered backprojection. For example, the first generation CT system has uniform ray density because the ray spacing is uniform. In contrast, the ray density of the third generation CT system is non-uniform due to the different data acquisition geometry, and the analytical expression for the ray density was developed [1]. However, it is very hard to find the analytical expression of the ray density for the new system. So, our goal is to estimate the ray density heuristically. Since the filtered backprojection can be expressed as one system matrix, we can expressed the reconstruction process as follows

$$y^i = A\theta^i \tag{1}$$

where, A is the system matrix for the reconstruction, y is the ideal reconstructed value(or training data set), and θ is a ray density vector that we want to estimate.

To find the ray density for the new CT system, the reconstruction steps, filtering and backprojection, are expressed as a matrix operation. For a projection with m samples, m ray densities and 2m-1 reconstruction filter coefficients are used. As a training data set, a centered-uniform-cylinder which is large enough for the ray density estimation was used so that the projection is the same for all views. We will then verify that the same ray density works for other objects.

The ray density multiplication and filtering for each fan beam can be conducted by the following matrix multiplication.

$$F_i = H(P_i \bullet \theta_i) = \begin{bmatrix} h_1 & 0 & \cdots & 0 & 0 \\ h_2 & h_1 & \cdots & \vdots & \vdots \\ h_3 & h_2 & \cdots & 0 & 0 \\ \vdots & h_3 & \cdots & h_1 & 0 \\ h_{2m-2} & \vdots & \cdots & h_2 & h_1 \\ h_{2m-1} & h_{2m-2} & \vdots & \vdots & h_2 \\ 0 & h_{2m-1} & \cdots & h_{2m-3} & \vdots \\ 0 & 0 & \cdots & h_{2m-2} & h_{2m-3} \\ \vdots & \vdots & \vdots & h_{2m-1} & h_{2m-2} \\ 0 & 0 & 0 & \cdots & h_{2m-1} \end{bmatrix} \begin{bmatrix} p_{1,i}\theta_{1,i} \\ p_{2,i}\theta_{2,i} \\ p_{3,i}\theta_{3,i} \\ . \\ . \\ . \\ . \\ . \\ . \\ p_{m,i}\theta_{m,i} \end{bmatrix} = \begin{bmatrix} h_1 p_{1,i} & 0 & \cdots & 0 & 0 \\ h_2 p_{1,i} & h_1 p_{2,i} & \cdots & \vdots & \vdots \\ h_3 p_{1,i} & h_2 p_{2,i} & \cdots & 0 & 0 \\ \vdots & h_3 p_{2,i} & \cdots & h_1 p_{m-1,i} & 0 \\ h_{2m-2} p_{1,i} & \vdots & \cdots & h_2 p_{m-1,i} & h_1 p_{m,i} \\ h_{2m-1} p_{1,i} & h_{2m-2} p_{2,i} & \vdots & \vdots & h_2 p_{m,i} \\ 0 & h_{2m-1} p_{2,i} & \cdots & h_{2m-3} p_{m-1,i} & \vdots \\ 0 & 0 & \cdots & h_{2m-2} p_{m-1,i} & h_{2m-3} p_{m,i} \\ \vdots & \vdots & \vdots & h_{2m-1} p_{m-1,i} & h_{2m-2} p_{m,i} \\ 0 & 0 & 0 & \cdots & h_{2m-1} p_{m,i} \end{bmatrix} \begin{bmatrix} \theta_{1,i} \\ \theta_{2,i} \\ \theta_{3,i} \\ . \\ . \\ . \\ . \\ . \\ . \\ \theta_{m,i} \end{bmatrix} = A_i \theta_i \tag{2}$$

where F_i is the filtered projection for the ith fan beam, P_i is the projection data for the ith fan beam, θ_i is the $m \times 1$ ray density for the ith fan beam, and H is a $(3m\text{-}2) \times m$ matrix of filter coefficients. A_i is a $(3m\text{-}2) \times m$ matrix whose terms contain filter coefficients and the ith fan beam data.

The final image is produced by summing the backprojections of each fan beam. Pixel-driven backprojection of each filtered projection using linear interpolation [2] can be described by the following equation.

$$R_{new\_CT} = \sum_{i=1}^{N_d} \left(\sum_{j=1}^{N_v} L_{ij} A_i \right) \theta_i \tag{3}$$

where N_d is the number of detectors, N_v is the number of views, L_{ij} contains the linear interpolation coefficients of the ith fan beam at the jth view, and R_{new_CT} is the reconstructed image.

In equation (3), L_{ij} and A_i are known for training data set, and θ_i is unknown. By comparing R_{new_CT} with the ideal training data set, R_{train}, the error vector E is defined as

$$E = R_{new\_CT} - R_{train} \tag{4}$$

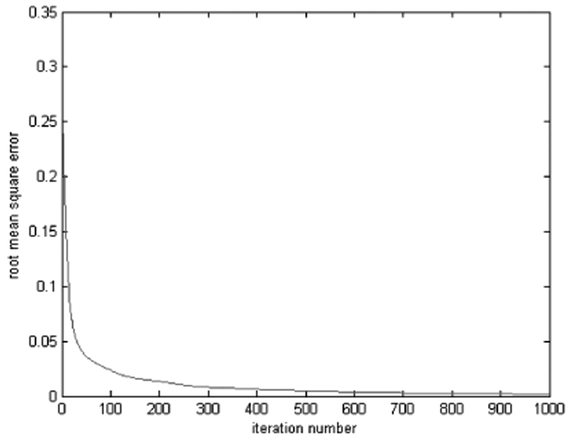
The ray density which minimizes the L_2 norm of E was found by using a conjugate-gradient method [3] , using $\mathbf{1}^{\mathrm{T}}$ as the initial guess. The root mean square (i.e., rms) error was calculated as in each iteration

$$rms_E = \frac{\|E\|_2}{\sqrt{n}} \tag{5}$$

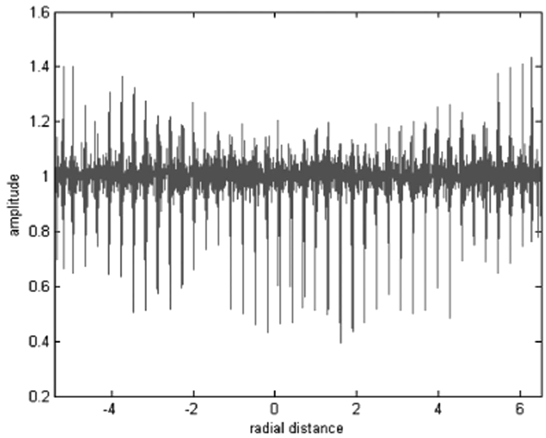
where, n is the length of the error vector E.

4. Result

The new CT system has 3200 rays per view, and therefore needs a ray density composed of 3200 values. Figure 2 (a) shows the rms error per iteration. The initial rms error was 0.2598, and after 1000 iterations it was reduced to 0.0019. Figure 2 (b) shows the ray density calculated using the training data set after 1000 iterations.



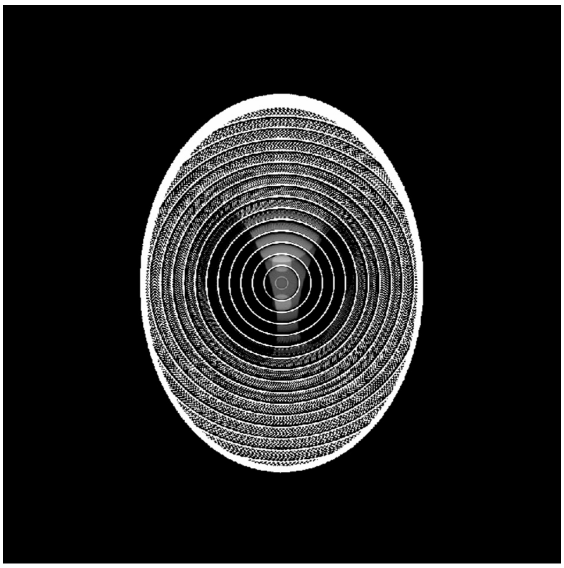
(a)



(b)

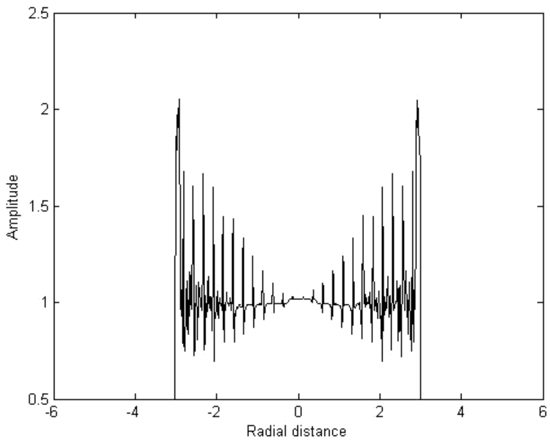
Figure 2. (a) rms error per iteration and (b) ray density for the new CT geometry

The Shepp-Logan phantom data [1] was reconstructed by using the ray density. Figures 3(a) and (b) shows the reconstructed images without and with applying ray density. Figures 3(c) and (d) plots the corresponding central profiles. Comparison with Figures 3 (a) and (b) shows that the ringing artifacts were suppressed significantly. This result supports the approach of calculating a ray density for a uniform centered cylinder (e.g, training data set) and applying it to an arbitrary object.

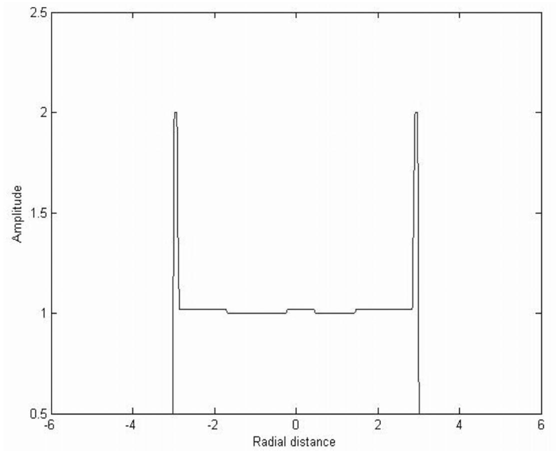


(a)

(b)



(c)



(d)

Figure 3. Reconstructed images and central profiles with and without applying the ray density

5. Conclusion

For the ray density estimation of a new CT system, supervised-learning algorithm was used. Since the ray density of a new CT system is very hard to derive analytically, heuristic method (Newton method by using a supervised-learning) was implemented. The estimated ray density from the training data set works well for general object, and the image quality was significantly improved after using the ray density. By adding some constraints to the ray density, we may expect the reduced variation of the ray density so that the noise pattern of a new CT system can be uniform.

Reference

[1] A. Kak and M. Slaney, “Principles of Computerized Tomographic Imaging,” *New York : IEEE Press,* 1988

[2] Jiang Hsieh, “Computed Tomography Principles, Design, Artifacts, and Recent Advances,” *Washington :SPIE Press,* 2003

[3] Michael T. Health, “Scientific Computing an Introductory survey,” *McGraw-Hill,* 2005